Numerical Mathematics syllabus

Course Description

This module helps you hone your skills in thinking abstractly. It also introduces you to many of the standard continuous models used to help understand and design computational systems. Through this module, you will develop the fundamental numerical mathematical tools that will support you throughout the BSc Computer Science programme. Particular attention is paid to notions of experimentation, reasoning, and generalization. You will learn a wide range of the numerical mathematical concepts and techniques that underpin Computer Science. In particular, you will study number systems, special functions, graphing and linear algebra.

Course Goals and Objectives

Upon successful completion of this course, you will be able to:

1. Transform numbers between number bases
2. Perform computations using trigonometric definitions and identities
3. Produce and interpret graphs of functions, including trigonometric, exponential and logarithmic graphs
4. Draw graphs of various special functions
5. Discover relationships between quantities by analyzing graphs
6. Translate between geometric and algebraic representations of spaces, including points, lines and linear transformations.

Textbook and Readings

The essentials readings for this course will come from the following text book, which is available in the University of London digital library:

- Bone, G., G. Chadha and N. Saunders *A Level Physics A for OCR Year 1 and AS student book*. (Oxford: OUP, 2015) PDF scan
- Croft, A. and R. Davison *Foundation maths.* (Harlow: Pearson, 2016) 6th edition. 9781292095196 Dawson ebook
- Larson, R. *Precalculus with limits*. (Boston, MA: Cengage, 2017) 4th edition. 9781337516853 Dawson ebook
- Yan, S.Y. *Number theory for computing*. (Berlin: Springer-Verlag, 2002) 2nd edition. PDF scan

- Vince, J. *Mathematics for computer graphics*. (London: Springer-Verlag, 2006) 2nd edition. Dawson ebook

This course does not require you to read the whole book, you will be given specific readings for each topic from these texts are listed with direct links on the Readings page for each topic.

You will also be asked to do some independent research from online sources or using the University of London digital library.

Course Outline

The course consists of 10 topics, each of which spans 2 weeks.

Topic 1: Number bases and modular arithmetic	Learning Objectives: • Explain the notion of a number base. • Carry out arithmetic operations in several bases. • Translate numbers between representations in different bases. • Give examples of the use of number systems in the context of computing. • Categorise numbers as rational or irrational when presented in digital expansion or as a fraction, and change between representations. • Perform modular arithmetic in different bases. • Give examples of the use of modular arithmetic in the context of computing.
Topic 2: Sequences and series	Learning Objectives: • Explain the notion of number sequence,

	arithmetic progression and geometric progression. • Explain and use the notion of series. • Given the general term or recurrence relation for a sequence, compute their terms. • Identify arithmetic and geometric progressions. • Give and use the formula and recurrence relation to describe and compute terms of arithmetic and geometric progressions. • Explain the notion of infinite sequence, convergent sequence and divergent sequence. • Give and use the formula to describe and compute the sum of finite or infinite geometric progressions.
Topic 3: Introduction to graph sketching and kinematics	Learning Objectives: • Explain and use Cartesian coordinates. • Produce a table of (x,y) values of a function y=f(x) that is polynomial, rational fraction or algebraic (involving radicals). • Specify the domain and range of a function. • Choose an adequate set of x-values to plot a graph, highlighting the main features: discontinuous domain, asymptotes, intercepts, maxima, minima and behaviour at infinity. • Choose an adequate scale for the graph.

	• Identify a function type given its graph. • Produce and use SUVAT equations to describe and compute speed, position, acceleration and time of a moving body.
Topic 4: Angles, triangles, trigonometry and beyond right-angled triangles	Learning Objectives: • Convert the amplitude of an angle between degrees and radians. • State and use the definition of radian. • Represent a triangle given information about the length of its edges and amplitude of its internal angles. • Describe and use the angle sum invariant to ascertain the feasibility of a triangle. • Give, recognise and use the triangular inequality to ascertain the feasibility of a triangle. • Give, recognise and use the definition of different types of angles and triangles. • State Pythagoras' Theorem and use it to calculate lengths of edges of a triangle.
Topic 5: Trigonometric functions and transformations of trigonometric graphs	Learning Objectives: • Determine the quadrant where a given angle or a given set of coordinates belongs to. • Map angles to points in the unit circle. • Compute trigonometric ratios of angles with amplitude outside the interval [0 degrees , 90 degrees] using trigonometric ratios of angles with amplitude within the interval [0 degrees , 90 degrees]. • Identify, produce graphs of and use

	trigonometric functions in degrees and radians. • Identify, describe and apply geometrical transformations (scaling, translation, reflection) to the graphs of sine, cosine and tangent functions. • Identify, describe and use key features of the trigonometric functions such as period, amplitude, maxima, minima and zeros. • Translate between Cartesian and polar coordinates.
Topic 6: Exponential functions; Logarithmic functions; inverses of exponential and logarithmic functions	Learning Objectives: • Compute by hand exponential expressions that involve integer numbers, fractions and radicals. • Compute exponential expressions using a calculator. • Use Euler's number e and give an approximation to 2 decimal places. • Identify, produce graphs of and use exponential functions. • Identify, describe and use key features of exponential functions such as domain, range, intercept, asymptote and growth. • Compute and produce the graph of the inverse of exponential and logarithmic functions. • Use geometrical transformations (scaling, translation, reflection) to sketch graphs of exponential and logarithmic functions and to solve equations.
Topic 7: Calculus: limits and differentiation; Understanding functions; further differentiation	Learning Objectives: • Calculate the limit of a function at a point using lateral limits, by using a sequence of values or simplifying the expression using division of polynomials or comparison to

	divergent or convergent sequences such as 1/n, 10^n, 0.1^n. • Calculate the limit of a function at infinity by using a sequence of values or comparison to divergent or convergent sequences such as 1/n, 10^n, 0.1^n or taking leading terms of rational expressions. • Use limits to identify asymptotes and the behaviour of the graph near them. • Formulate and calculate the differential of a function from first principles. • Calculate the differential of functions by using linearity, addition, multiplication, division and chain rules. • Use differentiation to find local maxima and minima, turning points in graphs of functions • Compute the differential of trigonometric, exponential, logarithmic functions.
Topic 8: Vectors and matrices; Inverting matrices, cross product of vectors	Learning Objectives: • Represent vectors algebraically: written as coordinates and as a linear combination of base vectors, and translate between the two representations. • Identify and calculate the magnitude, direction and orientation of a vector. • Perform linear combinations of vectors. • Translate between graphical and algebraic representations of vectors. • Calculate 'dot' product of 2D and 3D vectors and interpret the geometrical meaning. • Calculate the angle between two vectors. • Calculate 'cross' product of vectors and interpret geometrically.
Topic 9: Linear transformations and matrices; Affine transformations in	Learning Objectives: • Represent linear transformations as 2x2 matrices, given by a graphical representation or a geometric description.

homogeneous coordinates	- Describe the linear transformation (rotations, dilations, reflections and their combinations) associated to a given 2x2 matrix. - Use homogeneous coordinates to represent 2D points, vectors and matrices. - Compose transformations and identify non-commutative compositions. - Convert between simple coordinates and homogeneous coordinates, and simple matrices and homogenous matrices. - Solve a matrix equation of the form M v = w, where M is a square matrix and v and w are column matrices. - Rewrite a system of linear equations as a matrix equation involving 2x2 or 3x3 matrices.
Topic 10: Introduction to combinatorics and probability; Conditional probability; application	Learning Objectives: - Illustrate a counting problem by a tree diagram. - Use counting principles to calculate the size of an event and sample spaces - Distinguish between combinations and permutations. - Explain the number of permutations of length r chosen from a set of n objects. - Interpret the number of combinations of r objects chosen from a set of n distinct objects. - Compute conditional probability. - Use Bayes' Theorem and conditional probability to build a simple classifier for machine learning.

Learning Activities of This Course

The course is comprised of the following elements:

- Lecture videos. In each module the key concepts will be presented through a collection of short video lectures. You may stream these videos for playback within the browser by clicking on their titles or download the videos.
- Readings. Each topic may include several suggested readings. These are a core part of your learning, and, together with the videos, will cover all of the concepts you need for this course.
- Practice Quizzes. Each topic will include practice quizzes, intended for you to assess your understanding of the key concepts. End of topic quizzes in the second half of the course will not contribute to your final grade, but will provide important practice for your exam. You will be allowed unlimited attempts at each practice quiz. Each attempt may present a different selection of questions to you. There is no time limit on how long you take to complete each attempt at the quiz. All practice quizzes do not contribute toward your final score in the course.
- Graded Quizzes. Almost every week in topics 1-5 will include one summative quiz in the end of week. These quizzes will contribute to your coursework grade. You will be allowed 1 attempt per every 8 hours at each quiz. Your highest score will be used when calculating your final score in the summative quiz.
- Peer Reviewed Assignments. Topics 1-5 include peer reviewed assignments. These exercises will test your understanding of the concepts. You will read a short prompt, then perform a task in response. You will then be required to review three of your peers' submissions. Peer reviewed assignments will not contribute to your final grade, but will be valuable practice for your coursework and exam.
- Discussion Prompt. Each topic include discussion prompts. You will see the discussion prompt alongside other items in the lesson. Each prompt provides a space for you to respond. After responding, you can see and comment on your peers' responses. All prompts and responses are also accessible from your private group discussion forum.
- GeoGebra experimentations. In many lessons you will be provided with an opportunity to perform the tasks in GeoGebra plugins.

How to Pass This Course

The module has two major assessments each worth 50% of your grade:

- Coursework: this consists of several activities that you do on the Coursera platform and which will be assessed half way through course (after week 12)
- Written examination: you will take this at an examination centre in your country.

There are also several activities that are graded but have 0 weight. That means that they will not count towards your final grade, but they are a key part of your learning and you need to do them.

The mark shown on the Coursera platform is your coursework mark and you should remember that the exam counts for another 50%.

The coursework consists of several activities. This is a detailed breakdown of all of the marks.

Activity	Required?	Deadline week	Estimated time per module	% of final grade
End of week quizzes for topics 1-5	Yes	1-12	1-2 hours	25%
Peer reviews	Yes	10	15 hours	0%
Written, staff graded coursework	Yes	12	20 hours	25%
Written examination	Yes	22	2 hours 15 minutes	50%