1.2 Set Representation and Manipulation-Reading

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Cornell Notes	**Topic:** 1.2 Set Representation and Manipulation-Reading	Course: BSc Computer Science Class: Discrete Mathematics-Reading Date: October 17, 2019

Essential Question:

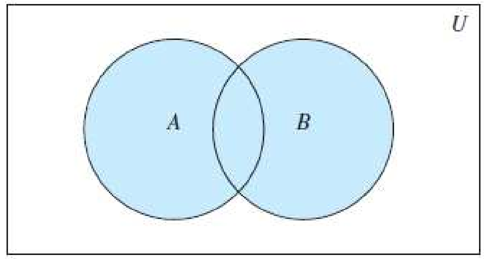
What is the various set operations that one can perform given sets and how does extend to a collection of sets?

Questions/Cues:

- What is the union of two sets?
- What is the intersection of two sets?
- What does it mean if two sets are disjoint?
- What is the principle of inclusion-exclusion?
- What is the set difference of two sets?
- What is the complement of a set?
- What is the relation between the difference of a set and the intersection between a set and a complement?
- What is the union of a collection of sets?
- What is the intersection of a collection of sets?
- What are the extended notations for a union and intersection of a collection of set when applied to another family of sets?

Notes

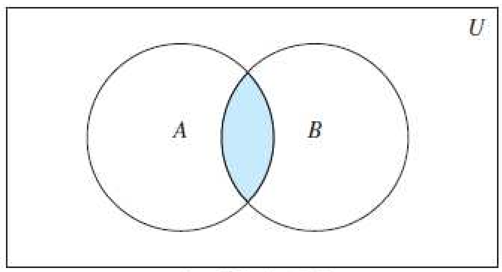
- Union of A and B = denoted by $A \cup B$, the set that contains elements that are either in A or in B, or in both
  - element x belongs to union of sets A and B ↔ x belongs to A or x belongs to B
  - $A \cup B = \{x \mid x \in A \vee x \in B\}$
    - $\vee$ = or



$A \cup B$ is shaded.

FIGURE 1 **Venn Diagram of the Union of A and B.**

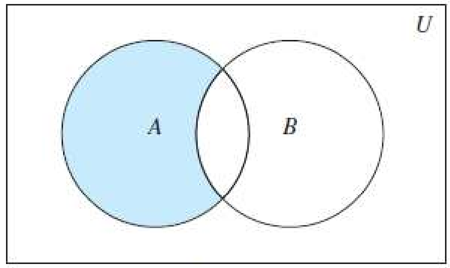
- Intersection of A and B = denoted by $A \cap B$, the set containing those elements in both A and B
    - element x belongs to intersection of A and B ↔ x belongs to A and x belongs to B
    - $A \cap B = \{x \mid x \in A \wedge x \in B\}$



$A \cap B$ is shaded.

FIGURE 2 **Venn Diagram of the Intersection of A and B.**

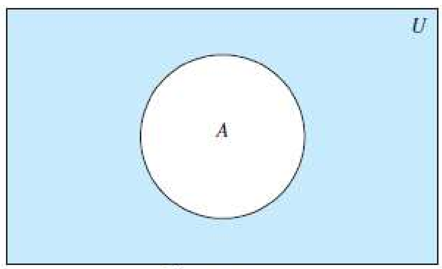
- Disjoint(Sets) = sets called disjoint if their intersection is empty set
- Principle of Inclusion-exclusion = used in finding cardinality of union of 2 finite A and B;
    - $|A| + |B|$ counts each element that in A but not in B ∨ in B but not in A exactly once and elements in both A and B exactly twice
    - To counteract we must subtract $|A \cap B|$ to count elements in intersection once
    - Hence, $|A \cup B| = |A| + |B| - |A \cap B|$
- Set Difference of A and B = denoted by A - B, the set containing those elements that in A but not in B
    - Difference of A and B also called complement of B with respect to A
    - sometimes denoted by A \ B
    - element x belongs to difference of A and B ↔ $x \in A$ and $x \notin B$.
    - $A - B = \{x \mid x \in A \wedge x \notin B\}$



$A - B$ is shaded.

FIGURE 3 **Venn Diagram for the Difference of A and B.**

- Let ***U*** be universal set, then complement of A = denoted by $\overline{A}$, is complement of A with respect to U
  - $\overline{A} = U - A$
  - element belongs to $\overline{A} \leftrightarrow x \notin A$
  - $\overline{A} = \{x \in U \mid x \notin A\}$



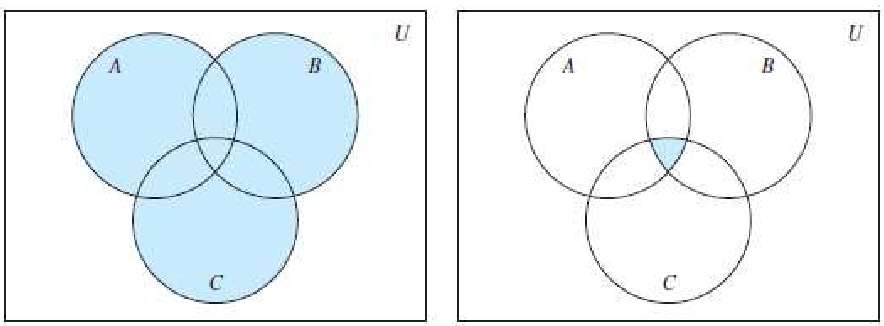
$\overline{A}$ is shaded.

FIGURE 4 **Venn Diagram for the Complement of the Set A.**

- $A - B = A \cap \overline{B}$

TABLE 1 Set Identities.

Identity	*Name*
$A \cap U = A$ $A \cup \emptyset = A$	Identity laws
$A \cup U = U$ $A \cap \emptyset = \emptyset$	Domination laws
$A \cup A = A$ $A \cap A = A$	Idempotent laws
$\overline{(\overline{A})} = A$	Complementation law
$A \cup B = B \cup A$ $A \cap B = B \cap A$	Commutative laws
$A \cup (B \cup C) = (A \cup B) \cup C$ $A \cap (B \cap C) = (A \cap B) \cap C$	Associative laws
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$	Distributive laws
$\overline{A \cap B} = \overline{A} \cup \overline{B}$ $\overline{A \cup B} = \overline{A} \cap \overline{B}$	De Morgan's laws
$A \cup (A \cap B) = A$ $A \cap (A \cup B) = A$	Absorption laws
$A \cup \overline{A} = U$ $A \cap \overline{A} = \emptyset$	Complement laws



(a) $A \cup B \cup C$ is shaded. (b) $A \cap B \cap C$ is shaded.

FIGURE 5 The Union and Intersection of A, B, and C.

- Union (Collection) = set that contains elements that are members of at least one set in collection

$$A_1 \cup A_2 \cup \cdots \cup A_n = \bigcup_{i=1}^{n} A_i$$

  - to denote the union of the sets $A_1, A_2, \ldots, A_n$.

- Intersection (Collection) = set that contains those elements that are members of all sets in collection

$$A_1 \cap A_2 \cap \cdots \cap A_n = \bigcap_{i=1}^{n} A_i$$

to denote the intersection of the sets $A_1, A_2, \ldots, A_n$.

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$$A_1 \cup A_2 \cup \cdots \cup A_n \cup \cdots = \bigcup_{i=1}^{\infty} A_i$$

to denote the union of the sets $A_1, A_2, \ldots, A_n, \ldots$.

$$A_1 \cap A_2 \cap \cdots \cap A_n \cap \cdots = \bigcap_{i=1}^{\infty} A_i.$$

- For set I, $\bigcap_{i \in I} A_i$ and $\bigcup_{i \in I} A_i$ = used to denote intersection and union of sets $A_i\, for\, i \in I$

Summary

In this week, we learned about the various set operations that can be performed on sets, the inclusion-exclusion principle, set identities to simplify set calculations and how the union and the intersection of set can be applied to a collection of sets and further extended to another family of sets.