2.1 Introduction to Functions-Reading

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Cornell Notes	**Topic:** 2.1 Introduction to functions-Reading	Course: BSc Computer Science Class: Discrete Mathematics-Reading Date: October 22, 2019

Essential Question:

What is a function and what are its properties?

Questions/Cues:

- What is a function?
- What is the domain, co-domain, range, image and pre-image of a function?
- How do we the sum and multiplication of two functions?
- How is an image of a subset of a set defined?
- What is a one-to-one function?
- What is an increasing or decreasing function?
- What is an onto function?
- What is a bijective function?
- What is the inverse of a function?

Notes

- Function = let A and B be nonempty set, function f from A to B is assignment of exactly 1 element of B to each element of A; write f (a) = b if b is unique element of B assigned by function f to element a of A.
    - If f is function from A to B, write $f: A \to B$
    - Functions sometimes also called mappings or transformations
- If function from A to B, A is domain of f and B is co-domain of f; if f (a) = b, b is image of a and a is pre-image of b. Range or image of f is set of all images of elements of A
    - If function f from A to B, say f maps A to B
    - 2 functions equal if have same domain, co-domain and map each elem. of common domain to same elem. in common co-domain
    - function called real-valued if co-domain is set of real numbers; called integer-valued if co-domain is set of integers
    - Two real-val or int-valued functions with same domain can added, well as multiplied
- Let f_1 and f_2 be function A to **R** = $f_1 + f_2$ and $f_1 f_2$ also functions from A to **R** defined for all $x \in A$ by:
    - $(f_1 + f_2)(x) = f_1(x) + f_2(x)$

- $(f_1f_2)(x) = f_1(x)f_2(x)$
- For function A to B, image of subset of A = image of S under function f is subset of B that consists of images of elements of S
  - image of S, f (S)
  - f (S) = { t | ∃s ∈ S (t = f (s)) }
  - shorthand, { f (s) | s ∈ S } to denote set
  - *** f (S) denotes set, not value of the function f for the set S
- one-to-one or an injunction of function f = if and only if f (a) = f (b) implies that a = b for all a and b in domain of f
  - function said to be injective if it's one-to-one
  - function f is one-to-one if and only if f (a) $\neq$ f (b) whenever a $\neq$ b
- increasing function = function f whose domain and codomain are subsets of set of real #, called increasing if $f(x) \leq f(y)$, strictly increasing if $f(x) < f(y)$, whenever x < y and x and y in the domain of f
- decreasing function = f called decreasing if $f(x) \geq f(y)$, strictly decreasing if $f(x) > f(y)$, whenever x < y and x and y in domain of f.
  - strictly = strict inequality
- onto or surjective function = function f from A to B is onto, if and only if for every element $b \in B$ there an element $a \in A$ with f (a) = b
  - function is onto if also range and co-domain are equal
- bijection = function f is one-to one correspondence or bijection, if both one-to-one and onto; said to be bijective
  - called invertible if inverse is definable, not invertible b/c no one-to-one correspondence and no inverse exists

Suppose that $f : A \to B$.

To show that f is injective Show that if $f(x) = f(y)$ for arbitrary $x, y \in A$ with $x \neq y$, then $x = y$.

To show that f is not injective Find particular elements $x, y \in A$ such that $x \neq y$ and $f(x) = f(y)$.

To show that f is surjective Consider an arbitrary element $y \in B$ and find an element $x \in A$ such that $f(x) = y$.

To show that f is not surjective Find a particular $y \in B$ such that $f(x) \neq y$ for all $x \in A$.

Let f be a one-to-one correspondence from the set A to the set B. The *inverse function* of f is the function that assigns to an element b belonging to B the unique element a in A such that $f(a) = b$. The inverse function of f is denoted by f^{-1}. Hence, $f^{-1}(b) = a$ when $f(a) = b$.

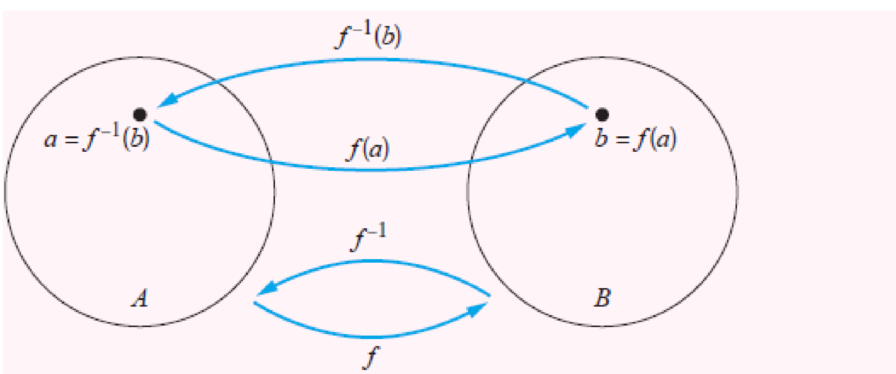

FIGURE 6 **The Function f^{-1} Is the Inverse of Function f.**

Summary

In this week, we learned what is function is, with what the domain, co-domain, and range of a function represent. Also we explored injective, surjective and bijective functions. Lastly, we looked at the invertibility of a function.