

## 2.2 More about Functions

**Notebook:** Discrete Mathematics [CM1020]

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### Cornell Notes

**Topic:**  
2.2 More about functions

Course: BSc Computer Science

Class: Discrete Mathematics-Lecture

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### Essential Question:

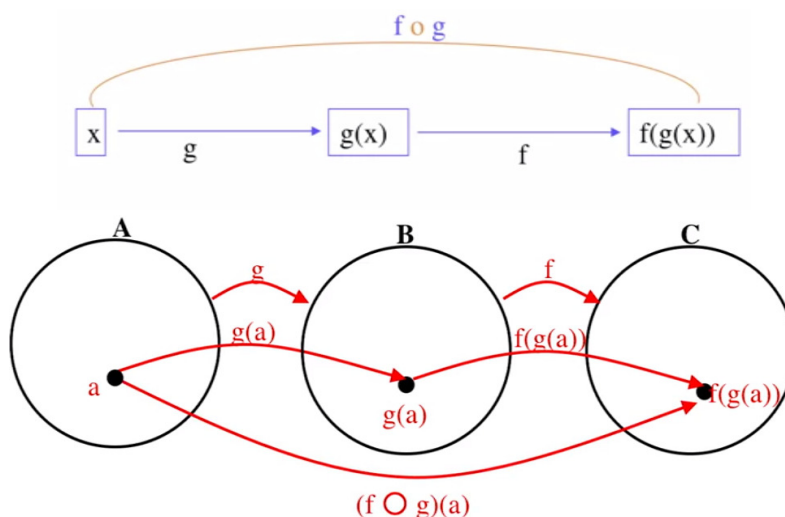
What is function composition, also what does it mean for a function to be bijective(invertible)? Alongside this, what are logarithmic, floor and ceiling functions?

### Questions/Cues:

- What is function composition?
- What does it mean when a function is bijective or invertible?
- What is the identity function in terms of function composition?
- What can be said about the graphs of  $f$  and its inverse?
- What is logarithmic function and what is its inverse?
- What are the laws of logarithms?
- What is the graph of the logarithmic functions and what are some of its properties?
- What is the floor function and its respective graph?
- What is the ceiling function and its respective graph?

### Notes

- Given 2 functions  $f$  and  $g$ ,  $(f \circ g)(x) = f(g(x))$



- Function composition is not commutative!  $f \circ g \neq g \circ f$ 
  - If we change order of  $f$  and  $g$ , we get different function
- Bijective or Invertible = if and only if it's both injective and surjective
  - Injective = one-to-one, for every  $x$  there is unique image in the co-domain
  - Surjective = onto, unique pre-image for each element in co-domain, and range = co-domain
- Inverse function = Let  $f: A \rightarrow B$ , if  $f$  is bijective (invertible), then inverse function  $f^{-1}$  exists and defined  $f^{-1}: B \rightarrow A$
- $(f \circ f^{-1})(x) = (f^{-1} \circ f)(x) = x$

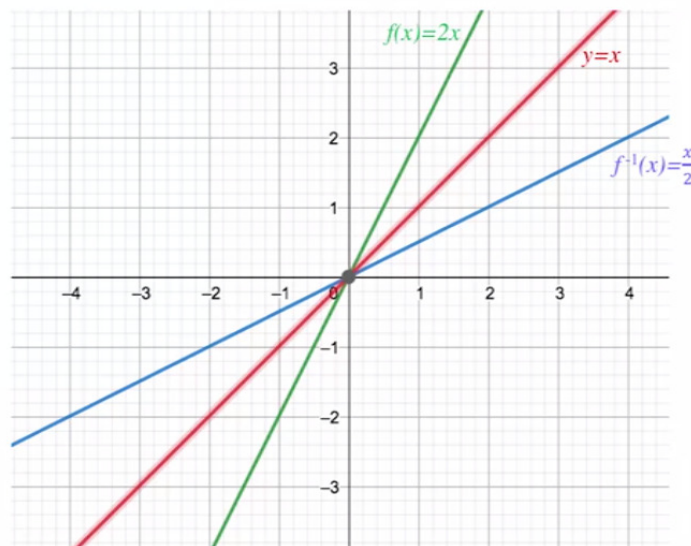
$$f: \mathbb{R} \rightarrow \mathbb{R} \quad \text{with } f(x) = 2x$$

$$f^{-1}: \mathbb{R} \rightarrow \mathbb{R} \quad \text{with } f^{-1}(x) = \frac{x}{2}$$

$$(f \circ f^{-1})(x) = f(f^{-1}(x)) = f\left(\frac{x}{2}\right) = 2 \cdot \frac{x}{2} = x$$

$$(f^{-1} \circ f)(x) = f^{-1}(f(x)) = f^{-1}(2x) = \frac{2x}{2} = x$$

The curves of  $f$  and  $f^{-1}$  are symmetric with respect to the straight line  $y = x$ .



- Logarithmic function = with base  $b$ ,  $b > 0$  and  $b \neq 1$  is defined:

$$\log_b x = y \quad \text{if and only if} \quad x = b^y$$

$\log_b x$  is the inverse function of the exponential function  $b^x$

$$\log_b m * n = \log_b m + \log_b n$$

$$\log_b \frac{m}{n} = \log_b m - \log_b n$$

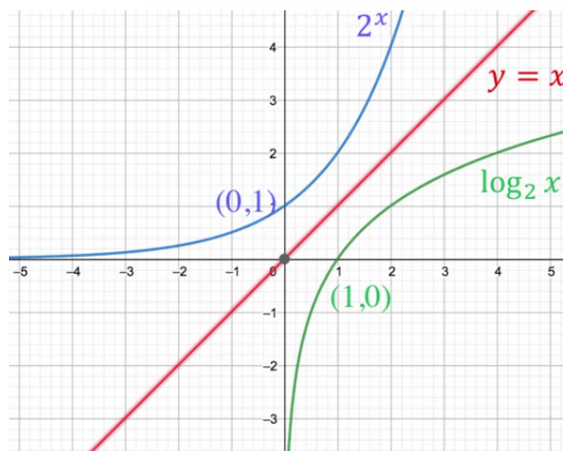
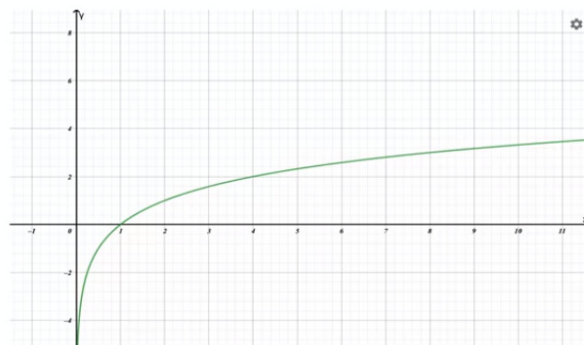
$$\log_b m^n = n \log_b m$$

$$\log_b 1 = 0$$

$$\log_b b = 1$$

Consider  $f(x) = \log_2 x$ . We will create a table of values for  $x$  and  $f(x)$  and then sketch a graph of  $f$ .

$x$	1/8	1/4	1/2	1	2	4	8
$f(x)=\log_2 x$	-3	-2	-1	0	1	2	3



Log properties:

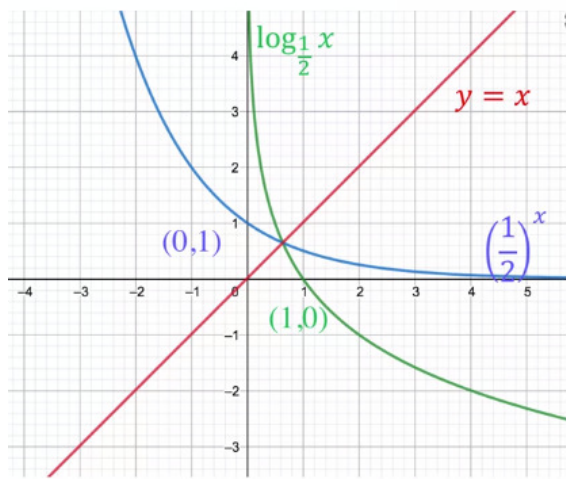
Graph of  $\log_2 x$  is symmetric to  $2^x$  with respect to  $y=x$

Domain:  $(0, \infty)$

Range:  $(-\infty, \infty)$

x-intercept:  $(1, 0)$

Increasing on:  $(0, \infty)$

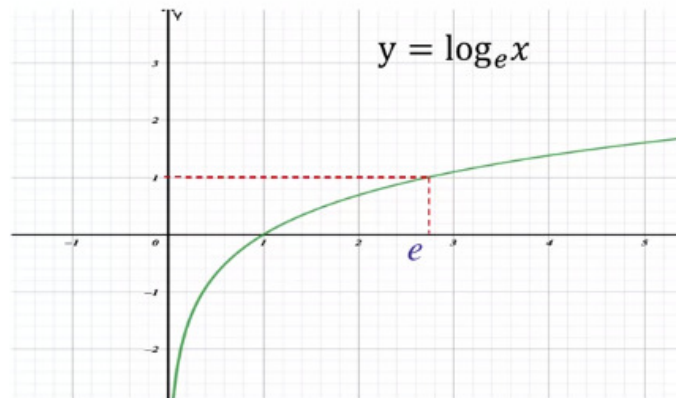


### Log Properties:

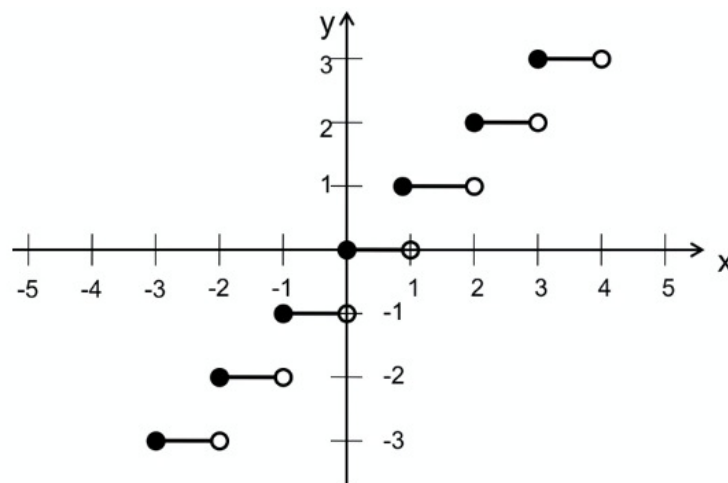
Graph of  $\log_{\frac{1}{2}} x$   
 is symmetric to  $\left(\frac{1}{2}\right)^x$   
 with respect to  $y=x$   
 Domain:  $(0, \infty)$   
 Range:  $(-\infty, \infty)$   
 x-intercept:  $(1, 0)$   
 Decreasing on:  $(0, \infty)$

$$\ln x = \log_e x \text{ where } e = 2.71828$$

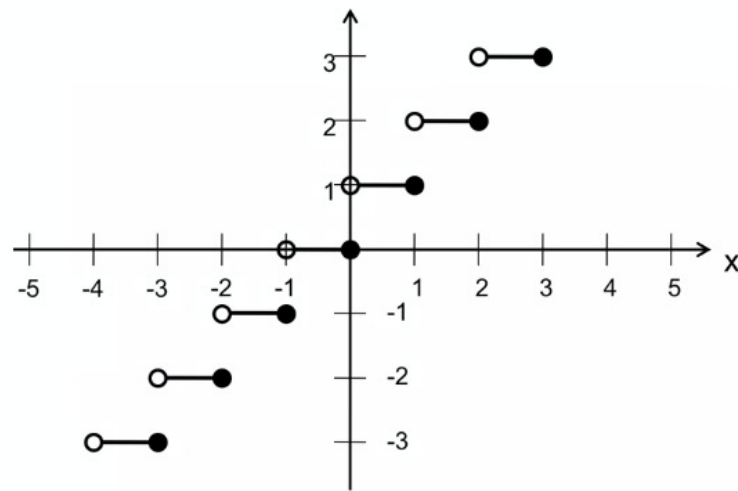
$$\ln e = \log_e e = 1$$



- Floor function = function  $\mathbb{R} \rightarrow \mathbb{Z}$  takes real number  $x$  as input, returns largest integer that is less than or equal to  $x$ , denoted  $\text{floor}(x) = \lfloor x \rfloor$ 
  - Floor of any integer is itself



- Ceiling function = function  $\mathbb{R} \rightarrow \mathbb{Z}$  takes real  $x$  as input, returns smallest integer that is greater than or equal to  $x$ , denoted  $\text{ceiling}(x) = \lceil x \rceil$ 
  - Ceiling of any integer is itself



Let  $n$  be an integer and  $x$  a real number. Show that:

$$\lfloor x+n \rfloor = \lfloor x \rfloor + n$$

**Proof:**

- Let  $m = \lfloor x \rfloor$
- hence,  $m \leq x < m+1$  (by definition)
- $m+n \leq x+n < m+n+1$
- this implies that  $\lfloor x+n \rfloor = m+n$  (by definition)
- hence,  $\lfloor x+n \rfloor = \lfloor x \rfloor + n$

### Summary

In this week, we learned what function composition is and what it means for a function to be bijective (invertible). Also, we looked at the logarithmic, floor and ceiling functions.