3.2 Applications

Notebook:	Discrete Mathematics [CM1020]		
Created:	2019-10-07 2:31 PM	**Updated:**	2019-11-07 12:04 PM
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Tags:	Contrapositive, Converse, Equivalence, Implication, Inverse, Precedence		

Cornell Notes	**Topic:** 3.2 Applications	Course: BSc Computer Science Class: Discrete Mathematics-Lecture Date: November 07, 2019

Essential Question:

What is an implication and/or what is equivalence and how to prove it?

Questions/Cues:

- What is an implication?
- What is the Converse, Contrapositive, and Inverse of an Implication?
- What is an equivalence statement?
- What does it mean if two propositions p & q are said to be logically equivalent?
- How do we determine non-equivalence?
- What is the order of precedence of the logical operators?
- What are laws of propositional logic to prove equivalence?
- What are the laws of propositional logic to prove equivalence in terms of negation?

Notes

- Implication (Conditional Statement) = Let p & q be propositions. Conditional statement $p \rightarrow q$ is proposition "if p then q"
  - p called hypothesis (or antecedent or premise)
  - q called conclusion (or consequence)

Let p and q be the following statements:

- p: "John did well in discrete mathematics."
- q: "John will do well in the programming course."

The conditional statement p → q can be written as follows:

"If John did well in discrete mathematics then John will do well in the programming course."

p	q	p → q
F	F	T
F	T	T
T	F	F
T	T	T

Correct reasoning: False premise, false or true conclusion

Incorrect reasoning: True premise, false conclusion

Correct reasoning: True premise, true conclusion

If the reasoning is correct (implication is true):
if the hypothesis is true then the conclusion is true

If the reasoning is incorrect (implication is false):
if the hypothesis is true, the conclusion is false

It's always true that:
from a false hypothesis any conclusion can be implied (true or false)

Let p and q be the following statements:

- p: "It's sunny."
- q: "John goes to the park."

<table>
<tr><td>p → q</td><td rowspan="9">If it's sunny then John goes to the park</td></tr>
<tr><td>if p then q</td></tr>
<tr><td>if p, q</td></tr>
<tr><td>p implies q</td></tr>
<tr><td>p only if q</td></tr>
<tr><td>q follows from p</td></tr>
<tr><td>p is sufficient for q</td></tr>
<tr><td>q unless ¬p</td></tr>
<tr><td>q is necessary for p</td></tr>
</table>

- Converse, Contrapositive, & inverse = Let p & q be propositions and A the conditional statement $p \rightarrow q$
    - Proposition $q \rightarrow p$ is the converse of A
    - Proposition $\neg q \rightarrow \neg p$ is the contrapositive of A
    - Proposition $\neg p \rightarrow \neg q$ is the inverse of A

Let p & q be the following statements:
p: "It's sunny"
q: "John goes to the park"
A the statement $p \rightarrow q$: "If it's sunny then John goes to the park"

The converse of A is : "If John goes to the park then it's sunny"
The contrapositive of A is: "If John doesn't go to the park then it's not sunny"
The inverse of A is: "If it's not sunny, then John doesn't go to the park"

Let p and q be two propositions concerning an integer n
- p: n has one digit
- q: n is less 10

1. First let's write the following statement using symbolic logic expression:
   "If the integer n has one digit then it is less than 10."
   $p \rightarrow q$

2. Now let's write its contrapositive using both symbolic logic expression and English:
   - $\neg q \rightarrow \neg p$
   - "If n is greater than or equal to 10 then n has more than one digit."

- Equivalence(Biconditional) = Let p & q be propositions. Biconditional or equivalence statement $p \leftrightarrow q$ is proposition "$p \rightarrow q$ & $q \rightarrow p$"
  - Biconditional statements also called bi-implications
  - $p \leftrightarrow q$ can also be read "p if and only if q"
  - Biconditional statement $p \leftrightarrow q$ true when p & q have same truth values; false otherwise

p	q	p → q	q → p	(p → q) ∧ (q → p): equivalence
F	F	T	T	T
F	T	T	F	F
T	F	F	T	F
T	T	T	T	T

- Let p & q be propositions. p & q are logically equivalent if they always have same truth value
  - We write $p \equiv q$
    - Symbol $\equiv$ is not logical operator, & $p \equiv q$ not compound statement but is the statement that $p \leftrightarrow q$ is always true

Example

Let's compare the two propositions $p \rightarrow q$ and $\neg p \vee q$

p	q	$p \rightarrow q$	$\neg p$	$\neg p \vee q$
F	F	T	T	T
F	T	T	T	T
T	F	F	F	F
T	T	T	F	T

The truth table shows that $\neg p \vee q$ is equivalent to $p \rightarrow q$ as they have the same truth values

- To determine non-equivalence, we can also use truth tables and find at least one row where truth values differ

Example

Let's examine whether the **converse** or the **inverse** of an implication is equivalent to the original implication.

p	q	$\neg p$	$\neg q$	$p \rightarrow q$	$\neg p \rightarrow \neg q$	$q \rightarrow p$
F	F	T	T	T	T	T
F	T	T	F	T	F	F
T	F	F	T	F	T	T
T	T	F	F	T	T	T

Operator	Precedence
$\neg$	1
$\wedge$	2
$\vee$	3
$\rightarrow$	4
$\leftrightarrow$	5

	Conjunction	Disjunction
idempotent laws	$p \vee p \equiv p$	$p \wedge p \equiv p$
commutative laws	$p \vee q \equiv q \vee p$	$p \wedge q \equiv q \wedge p$
associative laws	$(p \vee q) \vee r \equiv p \vee (q \vee r)$	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
distributive laws	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
identity laws	$p \vee \mathbf{F} \equiv p$	$p \wedge \mathbf{T} \equiv p$
domination laws	$p \vee \mathbf{T} \equiv \mathbf{T}$	$p \wedge \mathbf{F} \equiv \mathbf{F}$

Let's prove a case of distributive law: **p ∨ (q ∧ r) and (p ∨ q) ∧ (p ∨ r)**

p	q	r	q ∧ r	p ∨ (q ∧ r)	(p ∨ q)	(p ∨ r)	(p ∨ q) ∧ (p ∨ r)
F	F	F	F	F	F	F	F
F	F	T	F	F	F	T	F
F	T	F	F	F	T	F	F
F	T	T	T	T	T	T	T
T	F	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	T	F	F	T	T	T	T
T	T	T	T	T	T	T	T

	Conjunction	**Disjunction**
De Morgan's laws	¬(p ∨ q) ≡ ¬p ∧¬q	¬(p ∧ q) ≡ ¬p ∨ ¬q
absorption laws	p ∨ (p ∧ q) ≡ p	p ∧ (p ∨ q) ≡ p
negation laws	p ∨ ¬p ≡ **T**	p ∧ ¬p ≡ **F**
double negation law	¬¬p ≡ p	

Equivalence proof

Let's examine the equivalence between ¬(p ∧ (¬p ∨ q)) and (¬p ∨ ¬q):

¬(p ∧ (¬p ∨ q))	Given proposition
¬p ∨ ¬(¬p ∨ q)	*De Morgan's law*
¬p ∨ ((¬¬p) ∧ ¬q)	*De Morgan's law*
¬p ∨ (p ∧ ¬q)	*double negation law*
(¬p ∨ p) ∧ (¬p ∨ ¬q)	*distributive laws*
T ∧ (¬p ∨ ¬q)	*complement laws*
¬p ∨ ¬q	*identity laws*

Summary

In this week, we learned what an implication means and the implies operator is. Alongside this we looked at the converse, contrapositive and inverse of an implication, Biconditional statements, equivalence, logically equivalent propositions and how to prove equivalence using the laws of propositional logic. Finally, in the end we update our order of precedence for the logical operators to include implications and Biconditionals.