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Equality Comparison for Unordered Containers (Rev 1)

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Changes since last paper, n2944

- Updated all wording and section numbers to reflect the latest working paper n2960.
- Added missing member-qualifiers to many operations in the requirements tables.
- Mandated some documented requirements by using the term 'shall'.
- Add missing const-qualification to references in `operator==/!=` declarations.
- Removed discussion of algorithms with one range given by a single iterator. Deferred to a new LWG issue.
- As a matter of style, replaced "`!= false`" wordings with "`== true`".
- Revised wording for the `is_permutation` algorithm to clarify that `first2'` is an iterator marking the start of a range, rather than the range itself.
- Fixed numerous typos and fonts.

Background

Apart from the unordered containers, all of the container types in the current WP appear to be consistent with what N2479 refers to as having *value semantics*. In particular, each container type defines default construction, copy construction, copy assignment and the two (homogeneous, free) equality comparison operators, `operator==` and `operator!=`, with the truth of `operator==` being a postcondition of both copy construction and assignment (23.2.1 [container.requirements.general], Paragraph 5, Table 91 – Container requirements). By contrast, unordered containers are currently explicitly exempt (23.2.5 [unord.req], Paragraph 2, Table 97 – Container requirements that are not required for unordered associative containers, and Paragraph 10) from having to implement the equality comparison operators, yet in all other respects are consistent with having value semantics as defined by N2479.

This manifest inconsistency has been discussed in the past (circa 2004) and documented in N1837 (search for 6.2). The fundamental problem was that the equality function described in the container requirements (23.2.1 [container.requirements.general], Paragraph 5, Table 91 – Container Requirements) – i.e., that `==` is an equivalence relation and

```
a.size() == b.size() && equal(a.begin(), a.end(), b.begin())
```

and that the behavior is linear – “makes no sense for hash tables” because the order of iteration is not considered a *salient attribute* (i.e., one that contributes to value) of the container (see N2479) and separately because the (pathological) worst-case behavior of equality comparison is necessarily quadratic. The alternatives considered include: close as NAD; put in a caveat saying we don't quite satisfy the container requirements; put in the `operator==` defined in terms of `std::equal`; or put in Howard's (more useful) `operator==`. The second was chosen by a straw vote of 0-6-0-3.

Status Quo: Inconsistent and Incomplete

The unordered containers are, as defined today, inconsistent in that they are inherently value-semantic types that implement all of the value-semantic operations except for the equality comparison operations (`==` and `!=`). Moreover, the attempt to document the omission (23.2.5 [unord.req], Paragraph 2, Table 97 – Container requirements that are not required for unordered associative containers, and Paragraph 10) introduced additional inconsistencies, which (along with numerous other inaccuracies and inconsistencies) must be repaired, regardless. These unordered containers are also incomplete in that, unlike the ordered containers, they do not make explicit what abstract (“mathematical”) type they are approximating or what abstract (“externalizable”) values they are attempting to represent. Furthermore, these unordered containers omit an important, explicitly documented (23.2.1 [container.requirements.general], Paragraph 5, Table 91 – Container requirements) postcondition of all STL containers (and arguably all C++ *value-semantic*) types – i.e., that, after copy construction or copy assignment, the source and destination objects must compare equal i.e. they represent the same *value*. By defining equality comparison for unordered containers, we complete the set of *value-semantic* operations on them and, in so doing, force all of these *value-semantic* operations to be both self-consistent and also mutually consistent with their counterparts on the corresponding ordered containers.

Document Conventions

All section names and numbers are relative to the September 2009 working draft, N2960.

Existing working paper text is indented and shown in dark blue. Edits to the working paper are shown with ~~red strikeouts for deleted text~~ and green underlining for inserted text within the indented blue original text.

Comments and rationale mixed in with the proposed wording appears as shaded text.

Requests for LWG opinions and guidance appear with light (yellow) shading. It is expected that changes resulting from such guidance will be minor and will not delay acceptance of this proposal in the same meeting at which it is presented.

Discussion

Based on private discussions with various members of the LWG (as well as those documented in N1837), the impediments to defining homogeneous `operator==` and `operator!=` for unordered containers invariably involve one or more of the following concerns:

1. It is not clear we need them (let's not implement something that won't be used).
2. We don't know exactly how to define them (hence, we might get them wrong and have to live with another less-than-well-thought-out feature).
3. Any reasonable definition would come with a prohibitively expensive runtime cost (therefore any use would be subtly dangerous).

We will now address each of these concerns in turn.

Why do we need `operator==` for unordered containers?

Regarding the first concern (lack of utility), there are at least four separate reasons why we need equality comparison for unordered containers. The first and dominant reason is utility: The concept of value for unordered containers is meaningful and useful in practical applications. It is arguably more common to want to know if two sets have the same elements independent of order than it is to know if two sets have the same elements in the same order. For example, if we give a group of children bags of candy that allegedly *compare equal*, what's important is that, ultimately, the items in the bags respectively compare equal, not the order in which these items are initially discovered.

As a second example, consider implementing a (Hyper) Graph of N nodes in terms of STL containers. In standard matrix notation of a graph, a directed edge from Node i to j is present if there is a non-zero value in the (i, j) th element of the matrix. Two graphs have the same value if they have the same standard matrix representation. There are many ways we might consider using STL to model such an object. For example, we could use a `multimap<int, int>` of size E (number of edges) to encode each edge. The problem with such an approach is that it fails to implement the notion of value described in the specification, as the relative order of the edges emanating from a single node would be treated as a salient attribute of the object type and thus contribute to value. A better alternative would be to use an `unordered_multimap<int, int>`. Now the relative order of edges

emanating from a node will not be considered when comparing the two graphs (as required by the specification). Another correct approach would be to create a `vector<unordered_multiset<int> >` of size N to represent the (non-unique) set of adjacent edges at each O(1) randomly-accessible node. Yet another approach might be to create a `vector<multiset<int> >` of size N where the adjacent edges are kept ordered. In this case, the semantics for value happen to be correct because each member of each group of the equivalent keys in the (ordered) `multiset` represents the same value so the order in which they were added is not observable.

As a third and final example, consider implementing a `TelephoneBook` type whose value is defined solely by the data it contains and not by the order in which that data was added. Our initial choice might be a `map<string, int>`. This choice is fine assuming that the names are unique, but suppose there are occasionally duplicate names with distinct phone numbers (e.g., different people, or perhaps the same person with multiple numbers). In that case, we might consider using a `multimap<string, int>`, but now the order in which duplicate keys are added would necessarily affect value. If the order of entries with duplicate key values should not affect the overall value of a `TelephoneBook` object, then the data structure of choice is `unordered_multimap<string, int>`.

The second reason why we need equality comparison defined for unordered containers is consistency: There is simply no valid reason (see below) why unordered containers should be the only container types in the STL that do not support all of the essential *value-semantic* operations. Gratuitous inconsistency needlessly complicates understanding and creates barriers to effective use.

The third reason is to establish a common vocabulary: We want to provide a single uniform definition of value upon which everyone can rely. Without such a definition, developers will “roll their own” and these interpretations will invariably differ (e.g., perhaps some will conclude, based solely on implementation efficiency, that the order of equivalent keys should be considered part of the value).

The fourth reason is testability. Implementing this important, explicitly stated, postcondition of both copy construction and copy assignment allows developers who use such containers to unit-test their software using the standard “ $y == f(x)$ ” test paradigm – i.e., generate an initial (unordered container) value, x , apply an application subroutine, f , to x , separately generate an expected (unordered container) value, y , and finally use the provided (unordered container) equality-comparison operator, `==`, to assert that y has the same value as $f(x)$.

How should we define `operator==` for unordered containers?

Regarding the second concern (suboptimal specification), our extensive understanding of (and experience with) *value-semantic* types (as described in N2479) gives us a large body of knowledge including essential properties of *value semantics* that helps guide us in providing a useful, efficient, and consistent definition of *value* for unordered containers based on *salient attributes* – i.e., those aspects of an object’s state that contribute to its overall *value*. The salient attributes of an unordered

container are simply the values of the elements in the container, regardless of iteration order. Thus, two unordered containers have the same value if one is a permutation of the other.

In general, computing permutations is a quadratic operation. However, given two unordered containers that use the same hash and key-equivalence functions, the elements will be partitioned into key-equivalence groups that make comparison much more efficient. Thus, we define the result of `operator==` for unordered containers as follows: two unordered containers, **a** and **b**, have the same *value* if (1) `a.size() == b.size()`, and (2) for each contiguous group (ai_1, ai_2) of equivalent keys — as defined by `a.key_eq(k1, k2)` and returned by `a.equal_range(k1)` — in **a**, there exists a (necessarily unique) corresponding contiguous group (bi_1, bi_2) of equivalent keys — as defined by `b.key_eq(k1, k2)` and returned by `b.equal_range(k1)` — in **b** such that (i) `distance(ai_1, ai_2) == distance(bi_1, bi_2)` and there exists a reordering (bi'_1, bi'_2) of the elements in (bi_1, bi_2) such that `equal(ai_1, ai_2, bi'_1) == true`. Reference implementations consistent with both the above definition and the formal wording can be found in the appendix at the end of this document.

Wouldn't operator== for unordered containers be slow?

Regarding the third concern (prohibitive runtime cost), the useful and intuitive definition of equality comparison for unordered containers proposed here has a runtime cost that is linear in the average case and quadratic only in the pathological worst case. For the unordered containers that permit equivalent keys, the complexity of `operator==` is proportional to $\sum E_i^2$ in the average case and N^2 in the worse case, where N is `a.size()`, and E_i is the size of the i^{th} equivalent-key group in **a**. Note that if the number of duplicate keys (or even the maximum number of duplicates in any contiguous group of equivalent keys) is known to be bounded by a constant, then the overall (worst-case) cost of equality comparison is $O(a.size())$.

More generally, it should be understood that the expected behavior of nearly all of the operations on unordered containers (including copy construction and assignment, element insertion, and lookup) have an expected runtime behavior that is fast, but is a factor of N slower in the worst case; equality comparison is no different, and therefore deserves no special consideration in that regard. For unordered containers that support equivalent keys, allowing the container to be populated with largely duplicated keys, although likely to result in poor runtime performance, is not so much a fault of the data structure itself, but rather of its use by unenlightened clients (who would be well-advised to consider other strategies).

In conclusion, we have established that there is a need for equality comparison between unordered containers (of the same type), that a consistent and intuitive definition of such a (homogeneous) equality-comparison operation exists, and that its expected runtime is highly efficient in all but pathological cases — consistent with the vast majority of other operations on unordered containers. We therefore provide formal language to introduce the notion of value for unordered containers and, at the same time, clean up the numerous inaccuracies and inconsistencies we have found in section 23.2 [container.requirements], section 23.4 [associative], and especially section 23.5 [unord]. (Note that this

paper subsumes all of the defects and repairs noted in open issue #861.) Finally, an additional, general-purpose algorithm, `is_permutation` (analogous to `equal`), taking 3 iterators (with and without an optional binary predicate) is added to section 25.2 [alg.nonmodifying] in order to facilitate both the documentation and implementation of equality-comparison operators for unordered associative containers supporting non-unique keys.

Summary and Scope of Changes

In a nutshell, we propose to make the following changes to the WP:

- Add a new algorithm, `is_permutation`, that is analogous to `equal`.
- Define `operator==` and `operator!=` for unordered associative containers.
- Update container requirements tables.
- Repair defects in affected sections along the way as appropriate.

Proposed Wording

23.2.1 General Container Requirements [container.requirements.general]

Change Table 91 – Container Requirements as indicated:

<code>X u;</code>			post: <code>u.size()</code> <code>== 0</code> <code>u.empty()</code> <code>== true</code>	constant
<code>X()</code>			<code>X.size()</code> <code>== 0</code> <code>X().empty()</code> <code>== true</code>	constant
...				
<code>a == b</code>	convertible to bool	<code>==</code> is an equivalence relation. <code>a.size()</code> <code>b.size()</code> <code>distance(a.begin(), a.end())</code> <code>== distance(b.begin(), b.end())</code> && <code>equal(a.begin(), a.end(), b.begin())</code>	<code>T</code> is <code>EqualityComparable</code>	linear
...				
<code>a.size()</code>	<code>size_type</code>	<code>a.end()</code> <code>a.begin()</code> <code>distance(a.begin(), a.end())</code>		constant
<code>a.max_size()</code>	<code>size_type</code>	<code>size()</code> of the largest possible container		constant
<code>a.empty()</code>	convertible to bool	<code>a.size()</code> <code>== 0</code> <code>a.begin() == a.end()</code>		constant

These changes are to accommodate `std::forward_list`, which supports homogeneous equality comparison operators `operator==` and `operator!=`, but does not define a `size()` method (since it cannot be implemented in constant time). We considered adding an extra row for unordered container `operator==` to this table, but could not find a way to express the semantics in a way that was concise enough for the table format.

Change Table 93 – Allocator-Aware Container Requirements as indicated:

<code>X()</code> <code>X u;</code>	Requires: A is DefaultConstructible. constant post: <code>u.size() == 0</code> <code>u.empty() == true,</code> <code>u.get_allocator() == A()</code>
<code>X(m)</code> <code>X u(m);</code>	Post: <code>u.size() == 0</code> <code>u.empty() == true,</code> constant <code>u.get_allocator() == m</code>

23.2.3 General Container Requirements [sequence.reqmts]

Change Table 94 – Sequence container requirements (in addition to container) as indicated:

- Change the text in the Assertion/note column in the row for "`X(n, t) / X a(n, t)`" as follows:

[..] post: ~~`size()`~~`distance(a.begin(), a.end()) == n` [..]

- Change the Assertion/note column in the row for "`X(i, j) / X a(i, j)`" as follows:

[..] post: ~~`size()`~~`distance(a.begin(), a.end()) == distance(i, j)`
[..]

- Change the text in the Assertion/note column in the row for "`a.clear()`" as follows:

`a.erase(a.begin(), a.end())` post: ~~`size() == 0`~~`a.empty() == true`

23.2.4 Associative Containers [associative.reqmts]

Change Table 96 – Associative container requirements (in addition to container) as indicated:

Not every occurrence of `size()` was replaced, because all current associative containers have a `size()` member. The following changes ensure consistency regarding the semantics of `erase` for all tables, and add some missing objects.

- Change the text in the Complexity column in the row for "`a.insert(i, j)`" as follows:

$N \log(a.size() + N)$ (~~N is the distance from i to j~~) where $N == distance(i, j)$

- Change the text in the Complexity column in the row for "`a.erase(k)`" as follows:

`log(a.size()) + a.count(k)`

- c) Change the text in the Complexity column in the row for "`a.erase(q1, q2)`" as follows:

`log(a.size()) + N` where ~~N is the distance from $q1$ to $q2$~~ `== distance(q1, q2)`.

- d) Change the text in the Assertion/note column in the row for "`a.clear()`" as follows:

`a.erase(a.begin(), a.end())` post: ~~`size() == 0`~~ `a.empty() == true`

- e) Change the text in the Complexity column in the row for "`a.clear()`" as follows:

linear in `a.size()`

- f) Change the text in the Complexity column in the row for "`a.count(k)`" as follows:

`log(a.size()) + a.count(k)`

23.2.5 Unordered Associative Containers [unord.reqmsg

]

In section 23.2.5 [unord.req], change paragraph 2 and table 97 as follows:

- 2 Unordered associative containers conform to the requirements for Containers (23.2), except that the expressions `a == b` and `a != b` have different semantics than for the other container types and the expressions in table 97 are not required to be valid, where `a` and `b` denote values of a type `X`, and `X` is an unordered associative container class:

Table 97 – Container requirements that are not required for unordered associative containers

<code>a == b</code>
<code>a != b</code>
<code>a < b</code>
<code>a > b</code>
<code>a <= b</code>
<code>a >= b</code>

Change Paragraphs 5 and 6 as indicated:

- 5 Two values `k1` and `k2` of type `Key` are considered equal equivalent if the container's ~~equality~~ key equal function object returns `true` when passed those values. If `k1` and `k2` are ~~equal~~ equivalent, the hash function shall return the same value for both.
- 6 An unordered associative container supports *unique keys* if it may contain at most one element for each key. Otherwise, it supports *equivalent keys*. `unordered_set` and `unordered_map` support unique keys. `unordered_multiset` and `unordered_multimap` support equivalent keys. In containers that

support equivalent keys, elements with equivalent keys are adjacent to each other in the iteration order of the container. Thus, although the absolute order of elements in an unordered container is not specified, the elements are grouped into *equivalent-key groups*, such that all elements of each group have equivalent keys. Mutating operations on unordered containers shall preserve the relative order of elements within each equivalent-key group, unless otherwise specified. ~~For unordered_multiset and unordered_multimap, insert and erase preserve the relative ordering of equivalent elements.~~

Change Table 98 – Unordered associative container requirements (in addition to container) as indicated:

The same rationale as for Table 96 applies here plus we also correct the complexity of insert and the postcondition for `max_load_factor`.

- a. Change the text in the Complexity column in the row for "`a.insert(i,j)`" as follows:

Average case $O(N)$, where N is `distance(i,j)`. Worst case $O(N * (a.size() + N))$.
- b. Change the text in the Assertion/note column in the row for "`a.clear()`" as follows:

[.] Post: ~~`a.size() == 0`~~ `empty() == true`
- c. Change the text in the Assertion/note column in the row for "`a.max_load_factor(z)`" as follows:

Pre: z shall be positive. ~~Changes~~ May change the container's maximum load factor using z as a hint.

Change Paragraph 10 as indicated:

- ~~10 Unordered associative containers are not required to support the expressions `a == b` or `a != b`. [Note: This is because the container requirements define operator equality in terms of equality of ranges. Since the elements of an unordered associative container appear in an arbitrary order, range equality is not a useful operation. —end note]~~
- 10 Two unordered containers, a and b compare equal if `a.size() == b.size()` and, for every equivalent-key group $[Ea1, Ea2]$ obtained from `a.equal_range(Ea1)`, there exists an equivalent-key group $[Eb1, Eb2]$ obtained from `b.equal_range(Ea1)`, such that `distance(Ea1, Ea2) == distance(Eb1, Eb2)` and `is_permutation(Ea1, Ea2, Eb1) == true`. For `unordered_set` and `unordered_map`, the complexity of `operator==` (i.e., the number of calls to the `==` operator of the value type, to the predicate returned by `key_equal()` and to the hasher returned by `hash_function()`) is proportional to N in the average case, and to N^2 in the worst case, where N is `a.size()`. For `unordered_multiset` and `unordered_multimap`, the complexity of `operator==` is proportional to $\sum E_i^2$ in the average case, and to N^2 in the worst case, where N is `a.size()`, and E_i is the size of the i^{th} equivalent-key group in a . However, if the respective elements of each corresponding pair of equivalent-key groups, Ea_i and Eb_i , are arranged in the same order (as is

commonly the case, e.g., if `a` and `b` are unmodified copies of the same container), then the average-case complexity for `unordered_multiset` and `unordered_multimap` becomes proportional to N (but worst-case complexity remains $O(N^2)$ – i.e., for a pathologically bad hash function). The behavior of operators `==` and `!=` are undefined for unordered containers unless (1) the `Hash` and `Pred` function objects respectively have the same behavior for both containers, and (2) the equality comparison operator for `Key` is a refinement_x of the partition into equivalent-key groups produced by `Pred`.

Add a footnote X:

Equality comparison is a refinement of partitioning if no two objects that compare equal fall into different partitions.

The meaning of value for both ordered and unordered containers is based on the definition of value for the contained elements themselves, and not on that implied by `key_equal` as defined by the `pred` functor supplied to the container as a template interface policy. We have therefore elected to make explicit here the requirement (for overall container equality comparisons) that equality comparison be defined for contained elements.

We have made three additional explicit (compile-time) pre-conditions on the valid use of equality comparison for unordered containers, based on semantics, performance, and an absence of practical need to support the restricted behavior. The first restriction is that both containers have the same interpretation of equivalent keys. If they do not, then it is possible for two unique-key containers of the same C++ type that currently have the same value to be acted upon in the same way (e.g., via an insert of the same key value) and subsequently not have the same value, thus violating the fundamental property of value-semantic types. Although it is theoretically possible to retain proper value semantics and still allow unequal key comparators for non-unique unordered associative containers, the required algorithm would necessarily be quadratic in the average case.

The second restriction is that the hashers must have the same behavior. Allowing the hashers to be different would mean that we would give up an important optimization for both unique and non-unique unordered containers: When the number of buckets in the two unordered containers is the same, we can avoid having to hash each iterated element (or maintain its hash value along with its value) if, on lookup, we can assume that value hashes to the same bucket, which can be significant when the cost of hashing compared to that of equality comparison on elements is significant.

Finally, the requirement that equality comparison for keys be a refinement on key equivalence defined for unordered associative containers is also motivated by efficient implementation: Without this property, we cannot exploit the contiguous equivalent keys in non-unique associative containers, and will again have to resort to an algorithm that is quadratic even in the average case. While this restriction is necessary for the non-unique associative containers, relaxing it just for the unique ones seems to have insufficient practical benefit that would justify documenting the distinction.

Removing this third restriction for unordered containers with unique keys **only** is easy to do at the cost of a little more complexity in the description of the preconditions. Do people feel that such functionality would be useful?

23.5 Unordered associative containers [unord]

Add the following to the synopsis for `<unordered_map>`:

```
template <class Key, class T, class Hash, class Pred, class Alloc>
    bool operator==(const unordered_map<Key, T, Hash, Pred, Alloc>& a,
                    const unordered_map<Key, T, Hash, Pred, Alloc>& b);
template <class Key, class T, class Hash, class Pred, class Alloc>
    bool operator!=(const unordered_map<Key, T, Hash, Pred, Alloc>& a,
                    const unordered_map<Key, T, Hash, Pred, Alloc>& b);
template <class Key, class T, class Hash, class Pred, class Alloc>
    bool operator==(const unordered_multimap<Key, T, Hash, Pred, Alloc>& a,
                    const unordered_multimap<Key, T, Hash, Pred, Alloc>& b);
template <class Key, class T, class Hash, class Pred, class Alloc>
    bool operator!=(const unordered_multimap<Key, T, Hash, Pred, Alloc>& a,
                    const unordered_multimap<Key, T, Hash, Pred, Alloc>& b);
```

Add the following to the synopsis for `<unordered_set>`:

```
template <class Key, class T, class Hash, class Pred, class Alloc>
    bool operator==(const unordered_set<Key, T, Hash, Pred, Alloc>& a,
                    const unordered_set<Key, T, Hash, Pred, Alloc>& b);
template <class Key, class T, class Hash, class Pred, class Alloc>
    bool operator!=(const unordered_set<Key, T, Hash, Pred, Alloc>& a,
                    const unordered_set<Key, T, Hash, Pred, Alloc>& b);
template <class Key, class T, class Hash, class Pred, class Alloc>
    bool operator==(const unordered_multiset<Key, T, Hash, Pred, Alloc>& a,
                    const unordered_multiset<Key, T, Hash, Pred, Alloc>& b);
template <class Key, class T, class Hash, class Pred, class Alloc>
    bool operator!=(const unordered_multiset<Key, T, Hash, Pred, Alloc>& a,
                    const unordered_multiset<Key, T, Hash, Pred, Alloc>& b);
```

25 Algorithms Library [algorithms]

25.1 General [algorithms.general]

Add the following two declarations after those for `equal` but before those for `search`:

```
template<class ForwardIterator1, class ForwardIterator2>
    bool is_permutation(ForwardIterator1 first1, ForwardIterator1 last1,
                       ForwardIterator2 first2);
template <class ForwardIterator1, class ForwardIterator2,
          class BinaryPredicate>
    bool is_permutation(ForwardIterator1 first1, ForwardIterator1 last1,
                       ForwardIterator2 first2, BinaryPredicate pred);
```

25.2.12 Search [alg.search]

Insert a new section 25.2.12 [alg.is_permutation] after 25.2.11 [alg.equal] making Search section 25.2.13:

25.2.12 Is permutation [alg.is_permutation]

```
template<class ForwardIterator1, class ForwardIterator2>
    bool is_permutation(ForwardIterator1 first1, ForwardIterator1 last1,
                        ForwardIterator2 first2);
```

```
template<class ForwardIterator1, class ForwardIterator2,
        class BinaryPredicate>
    bool is_permutation(ForwardIterator1 first1, ForwardIterator1 last1,
                        ForwardIterator2 first2, BinaryPredicate pred);
```

- 1 Returns: true if there exists a permutation of the elements of the range `[first2, first2 + (last1 - first1))`, beginning with `ForwardIterator2 first2'`, such that `equal(first1, last1, first2')`, `equal(first1, last1, first2', pred) == true`. Otherwise, returns false.
- 2 Complexity: Exactly `distance(first1, last1)` applications of the corresponding predicate if `equal(first1, last1, first2)`, `equal(first1, last1, first2, pred)` would return true; otherwise at worst $O((distance(first1, last1))^2)$ applications of the corresponding predicate.

Acknowledgements

Daniel Krugler and Alisdair Meredith provided many helpful comments and alternate wording suggestions.

References

[N2960](http://www.open-std.org/JTC1/SC22/WG21/docs/papers/2009/n2960.pdf): Working Draft, Standard for Programming Language C++ (<http://www.open-std.org/JTC1/SC22/WG21/docs/papers/2009/n2960.pdf>)

[N2479](http://www.open-std.org/JTC1/SC22/WG21/docs/papers/2007/n2479.pdf): Normative Language to Describe Value Copy Semantics (<http://www.open-std.org/JTC1/SC22/WG21/docs/papers/2007/n2479.pdf>)

[N1837](http://www.open-std.org/JTC1/SC22/WG21/docs/papers/2005/n1837.pdf): Library Extension Technical Report – Issues List (<http://www.open-std.org/JTC1/SC22/WG21/docs/papers/2005/n1837.pdf>)

Appendix: Reference Implementations

The five code examples in this section illustrate defined behavior along with some useful optimizations.

Equality Comparison: `unordered_set`

```
template <class KEY>
bool operator==(const unordered_set<KEY>& a,
                const unordered_set<KEY>& b)
{
    typedef typename unordered_set<KEY>::const_iterator ConstIter;

    if (a.size() != b.size()) {
        return false;
    }
}
```

```

    for (ConstIter itr1 = a.begin(); itr1 != a.end(); ++itr1) {
        ConstIter itr2 = b.find(*itr1);
        if (itr2 == b.end() || *itr1 != *itr2) {
            return false;
        }
    }
    return true;
}

```

Equality Comparison: unordered_map

```

template <class KEY, class VALUE>
bool operator==(const unordered_map<KEY, VALUE>& a,
                const unordered_map<KEY, VALUE>& b)
{
    typedef typename unordered_map<KEY, VALUE>::const_iterator ConstIter;

    if (a.size() != b.size()) {
        return false;
    }

    for (ConstIter itr1 = a.begin(); itr1 != a.end(); ++itr1) {
        ConstIter itr2 = b.find(itr1->first);
        if (itr2 == b.end() || *itr1 != *itr2) {
            return false;
        }
    }
    return true;
}

```

Equality Comparison: unordered_multiset

```

template <class KEY>
bool operator==(const unordered_multiset<KEY>& a,
                const unordered_multiset<KEY>& b)
{
    typedef typename unordered_multiset<KEY>::const_iterator ConstIter;

    if (a.size() != b.size()) {
        return false;
    }

    for (ConstIter itr1 = a.begin();
         itr1 != a.end();
         /* Increment within the loop */) {

        // First check the two equivalent-key groups have the same size.

        pair<ConstIter, ConstIter> aRange = a.equal_range(*itr1);
        pair<ConstIter, ConstIter> bRange = b.equal_range(*itr1);

        if (distance(aRange.first, aRange.second)
            != distance(bRange.first, bRange.second)) {

```

```

        return false;
    }

    // Then check whether the two equivalent-key groups are permutations
    // of each other.

    if (!is_permutation(aRange.first,
                        aRange.second,
                        bRange.first)) {
        return false;
    }

    // Increment the iterator to the next equivalent-key group.

    itr1 = aRange.second;
}
return true;
}

```

Equality Comparison: unordered_multimap

```

template <class KEY, class VALUE>
bool operator==(const unordered_multimap<KEY, VALUE>& a,
               const unordered_multimap<KEY, VALUE>& b)
{
    typedef typename unordered_multimap<KEY, VALUE>::const_iterator ConstIter;

    if (a.size() != b.size()) {
        return false;
    }

    for (ConstIter itr1 = a.begin();
         itr1 != a.end();
         /* Increment within the loop */) {

        // First check the two equivalent-key groups have the same size.

        pair<ConstIter, ConstIter> aRange = a.equal_range(itr1->first);
        pair<ConstIter, ConstIter> bRange = b.equal_range(itr1->first);

        if (distance(aRange.first, aRange.second)
            != distance(bRange.first, bRange.second)) {
            return false;
        }

        // Then check whether the two equivalent-key groups are permutations
        // of each other.

        if (!is_permutation(aRange.first,
                            aRange.second,
                            bRange.first)) {
            return false;
        }
    }
}

```

```

        // Increment the iterator to the next equivalent-key group.

        itr1 = aRange.second;
    }
    return true;
}

```

Is_permutation

```

template <class ForwardIterator1, class ForwardIterator2>
bool
is_permutation(ForwardIterator1 first1, ForwardIterator1 last1,
               ForwardIterator2 first2)
{
    typedef typename
        iterator_traits<ForwardIterator2>::difference_type
        distance_type;

    // Efficiently compare identical prefixes: O(N) if sequences
    // have the same elements in the same order.

    for ( ; first1 != last1; ++first1, ++first2) {
        if (! (*first1 == *first2))
            break;
    }
    if (first1 == last1)
        return true;

    // Establish last2 assuming equal ranges by iterating over the
    // rest of the list.

    ForwardIterator2 last2 = first2;
    advance(last2, distance(first1, last1));

    for (ForwardIterator1 scan = first1; scan != last1; ++scan) {
        if (scan != find(first2, last2, *scan))
            continue; // We've seen this one before.
        distance_type matches = count(first2, last2, *scan);
        if (0 == matches || count(scan, last1, *scan) != matches)
            return false;
    }

    return true;
}

```