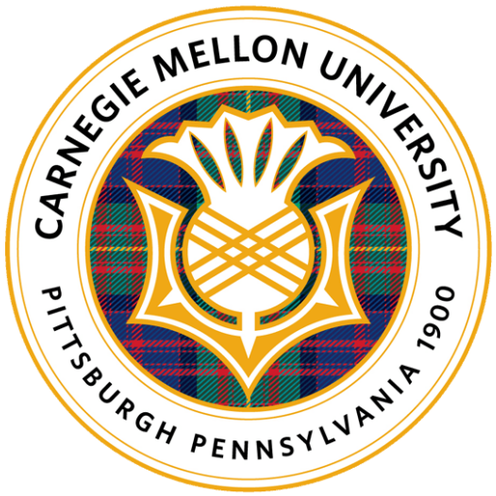
Verified Linear Session-Typed Concurrent Programming

Ankush Das*

Frank Pfenning

Carnegie Mellon University



PPDP 2020

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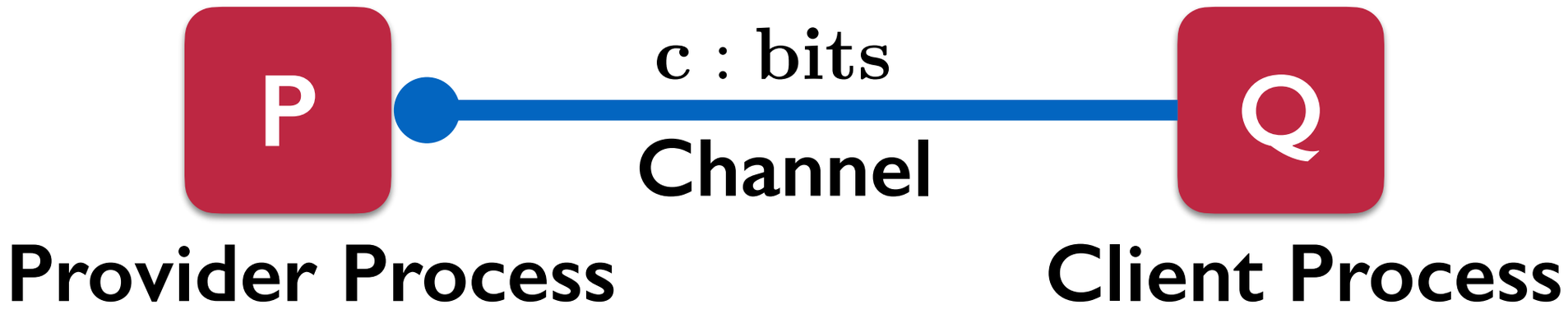

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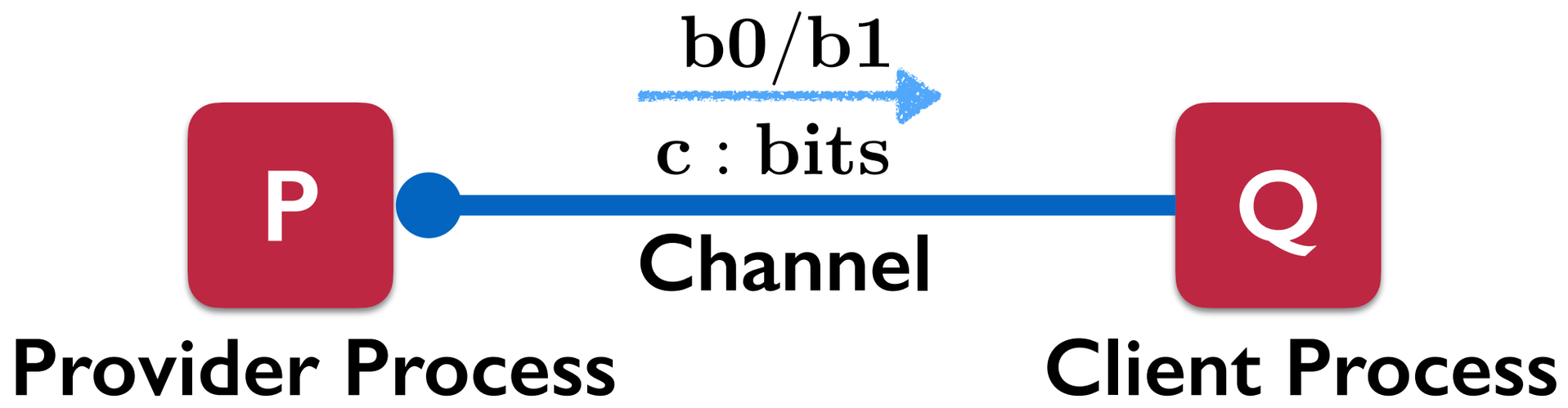

Example: Queues

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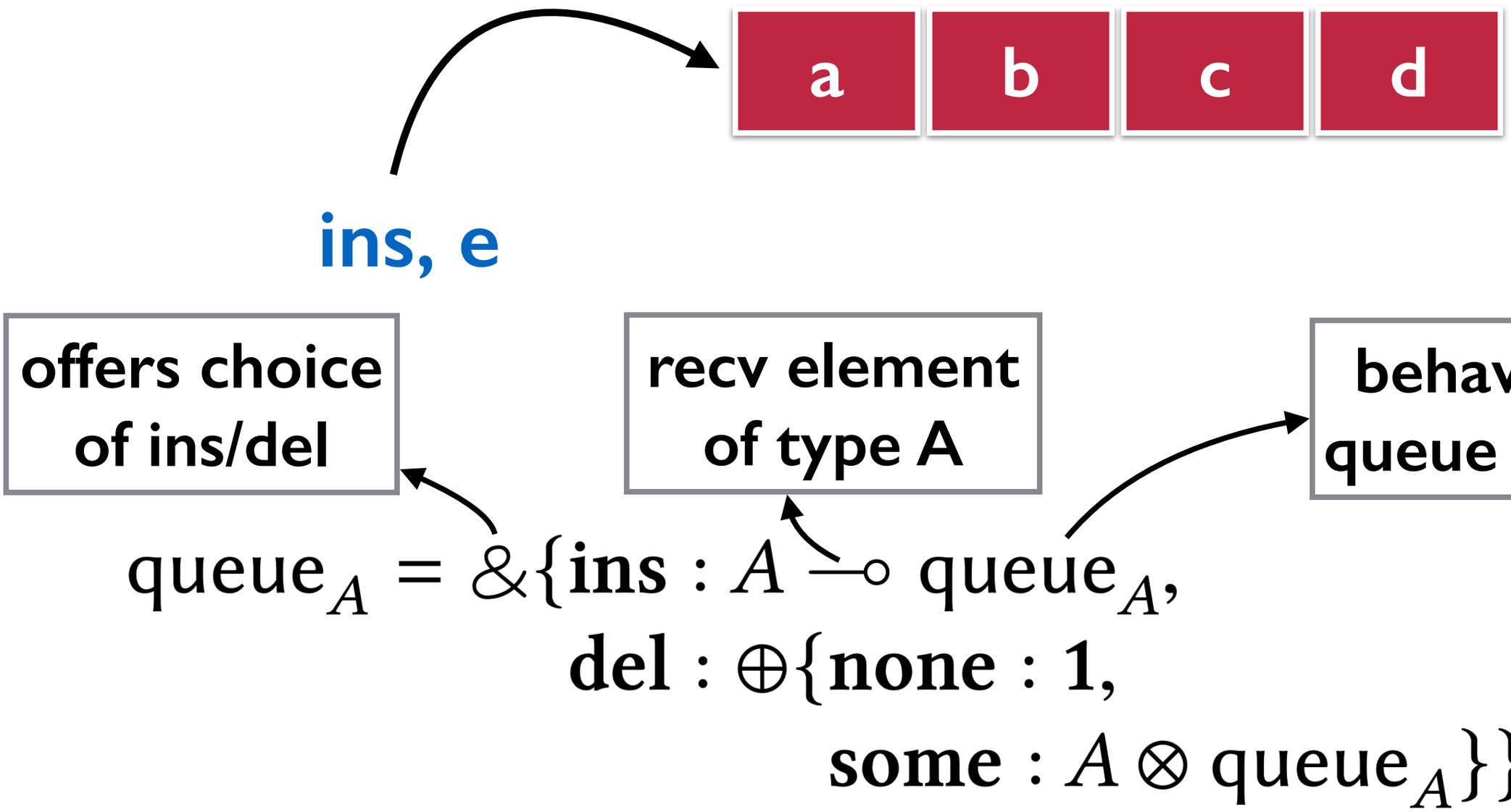

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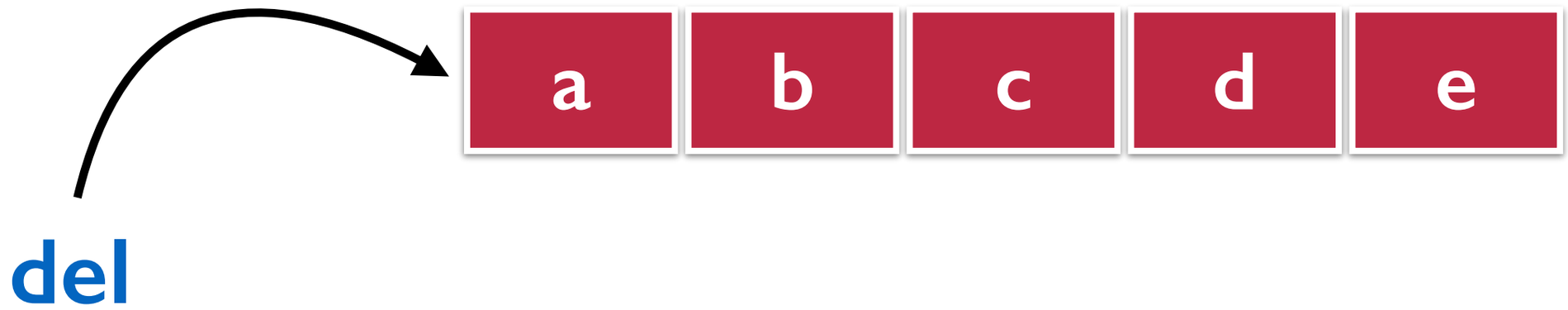

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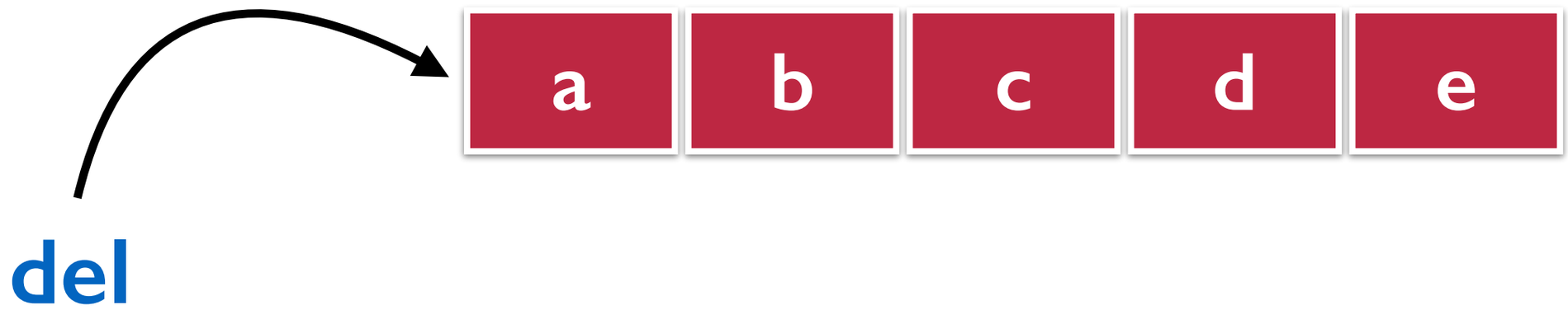

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terminate

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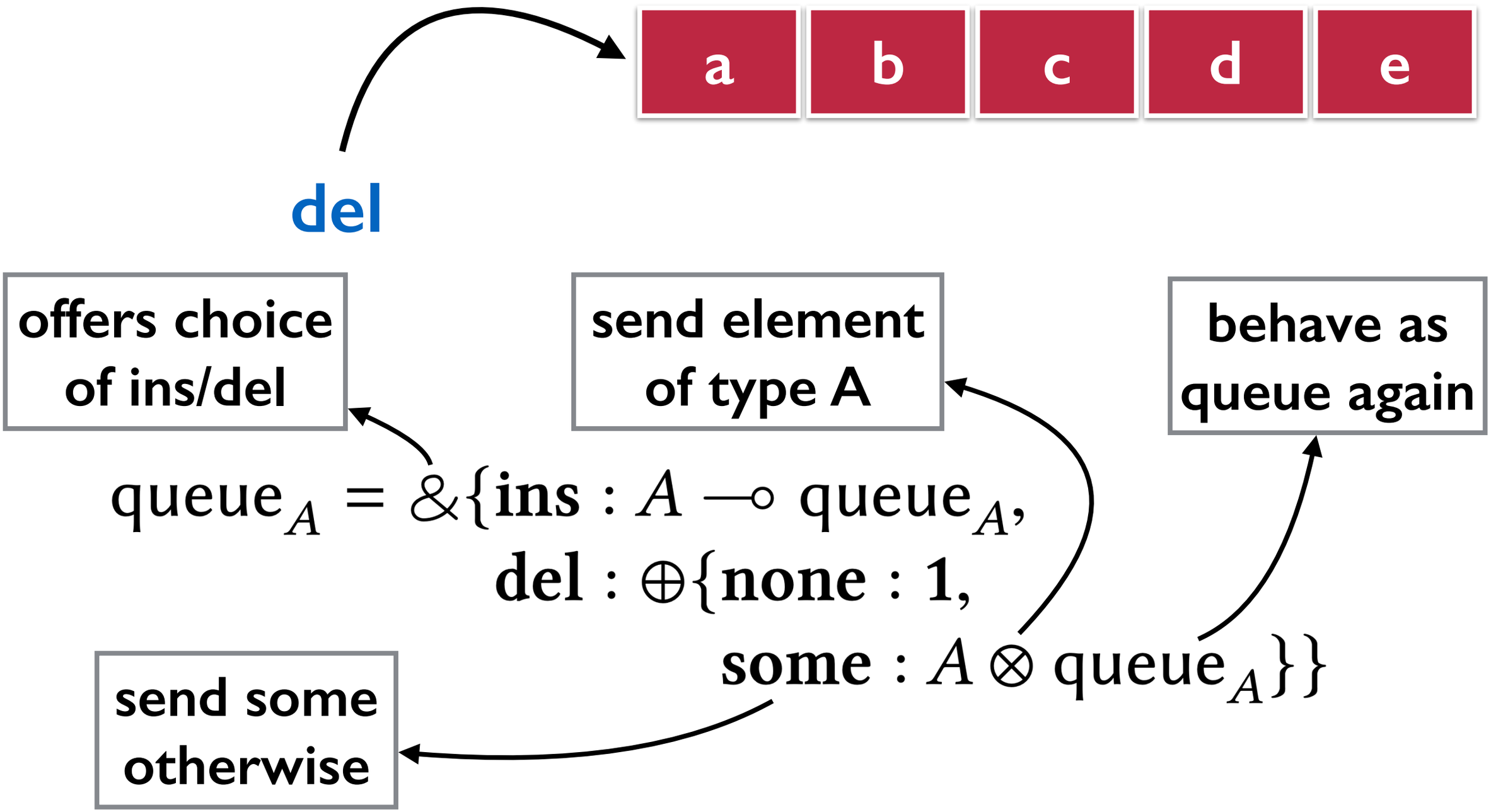

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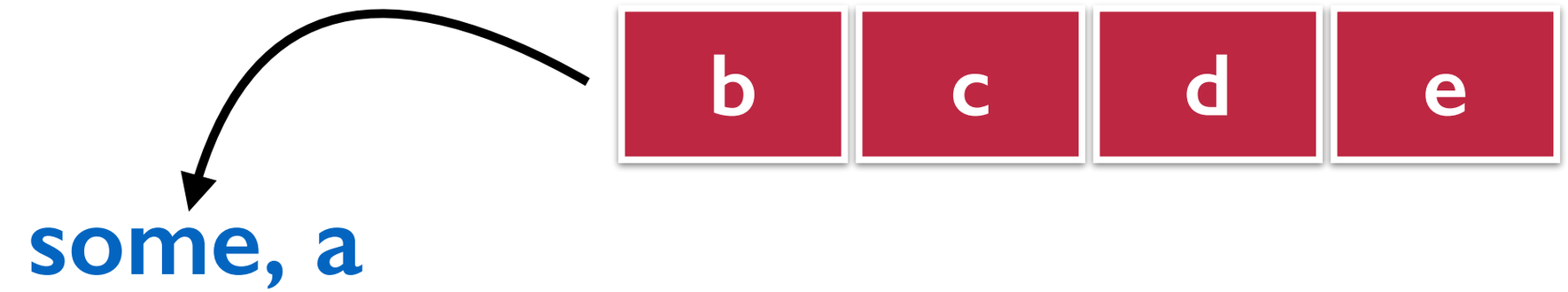

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When are the none and some branches taken?
Can the size of queue be encoded in the type?

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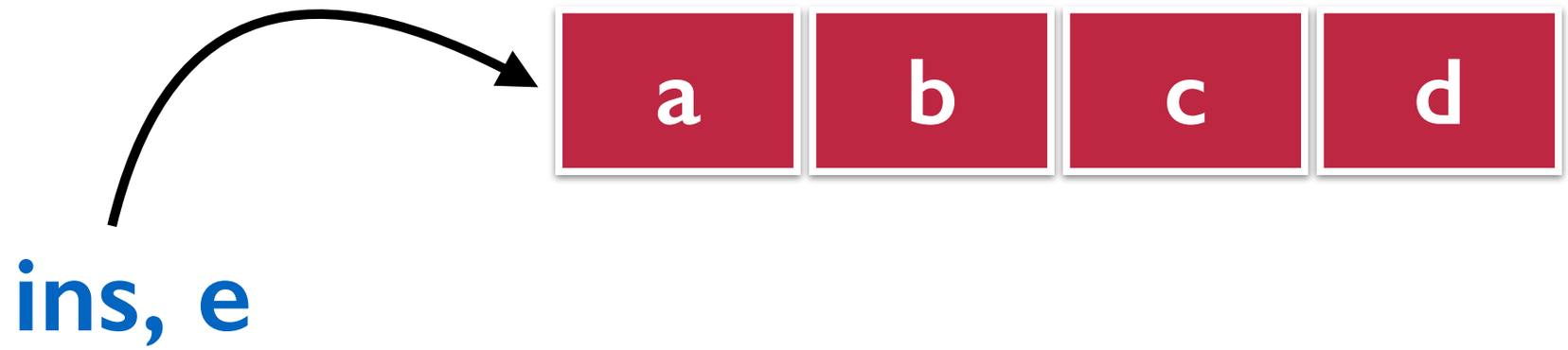

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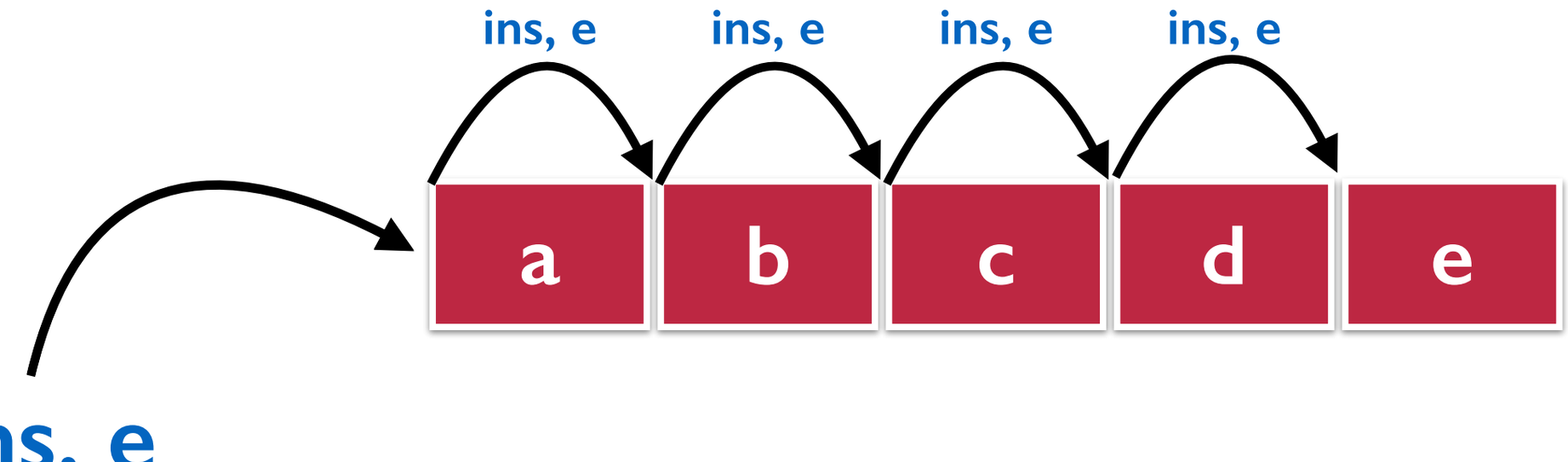

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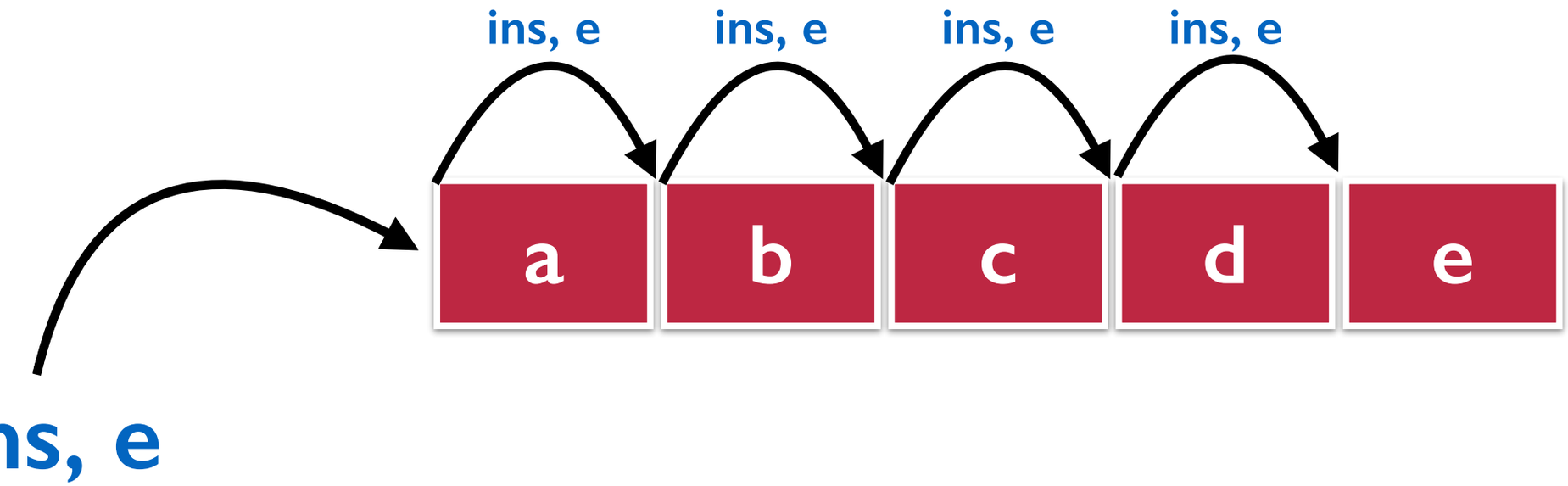

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Insertion: How do you refer to n in the queue type?

Refined Queue Type

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Proof Constraints (Sent by queue)

- ‘none’ branch: send (proof of) constraint $\{n=0\}$
- ‘some’ branch: send (proof of) constraint $\{n>0\}$
- Domain of constraints: Presburger Arithmetic

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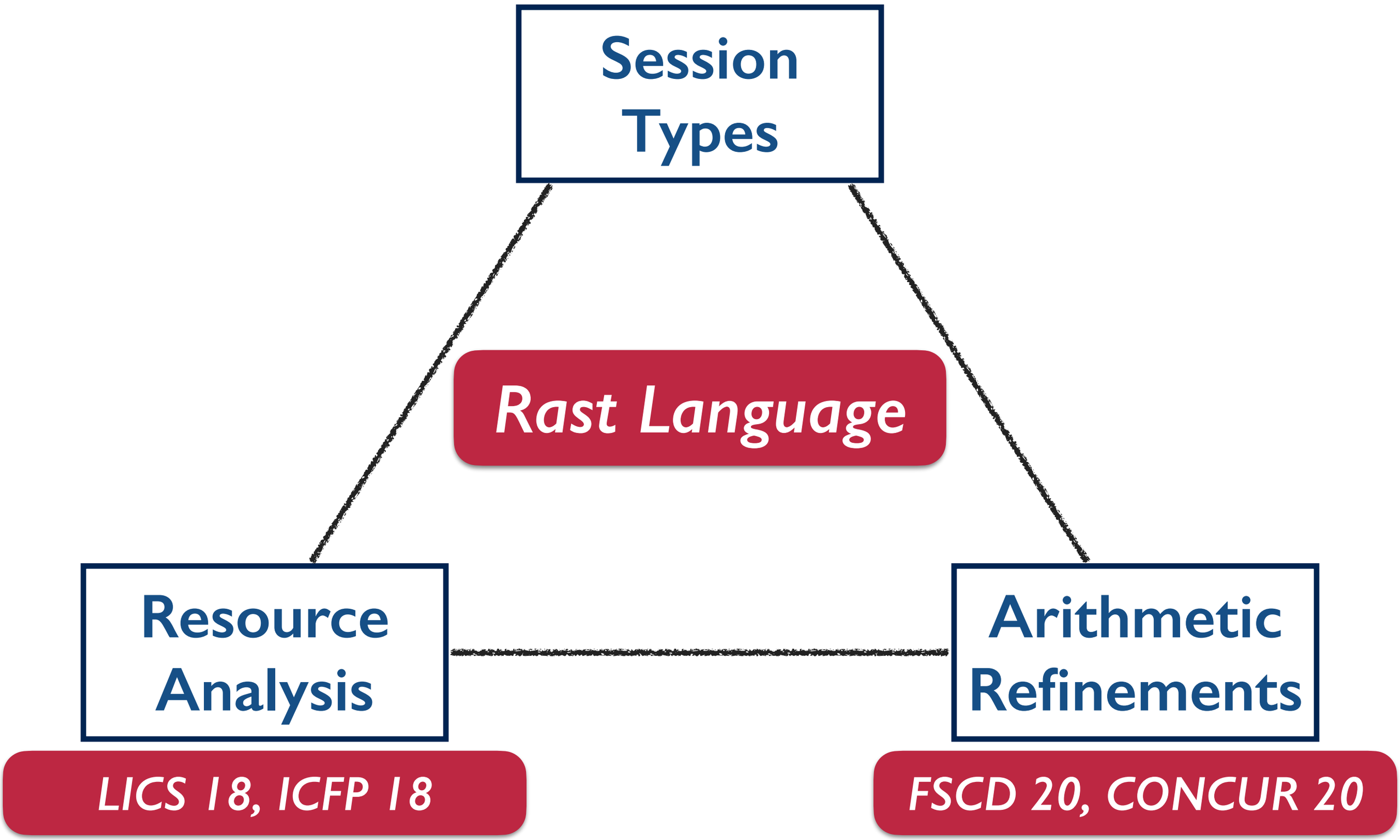
Since Presburger arithmetic is decidable and only closed programs are executed, no need to exchange proofs/constraints at runtime

Contributions



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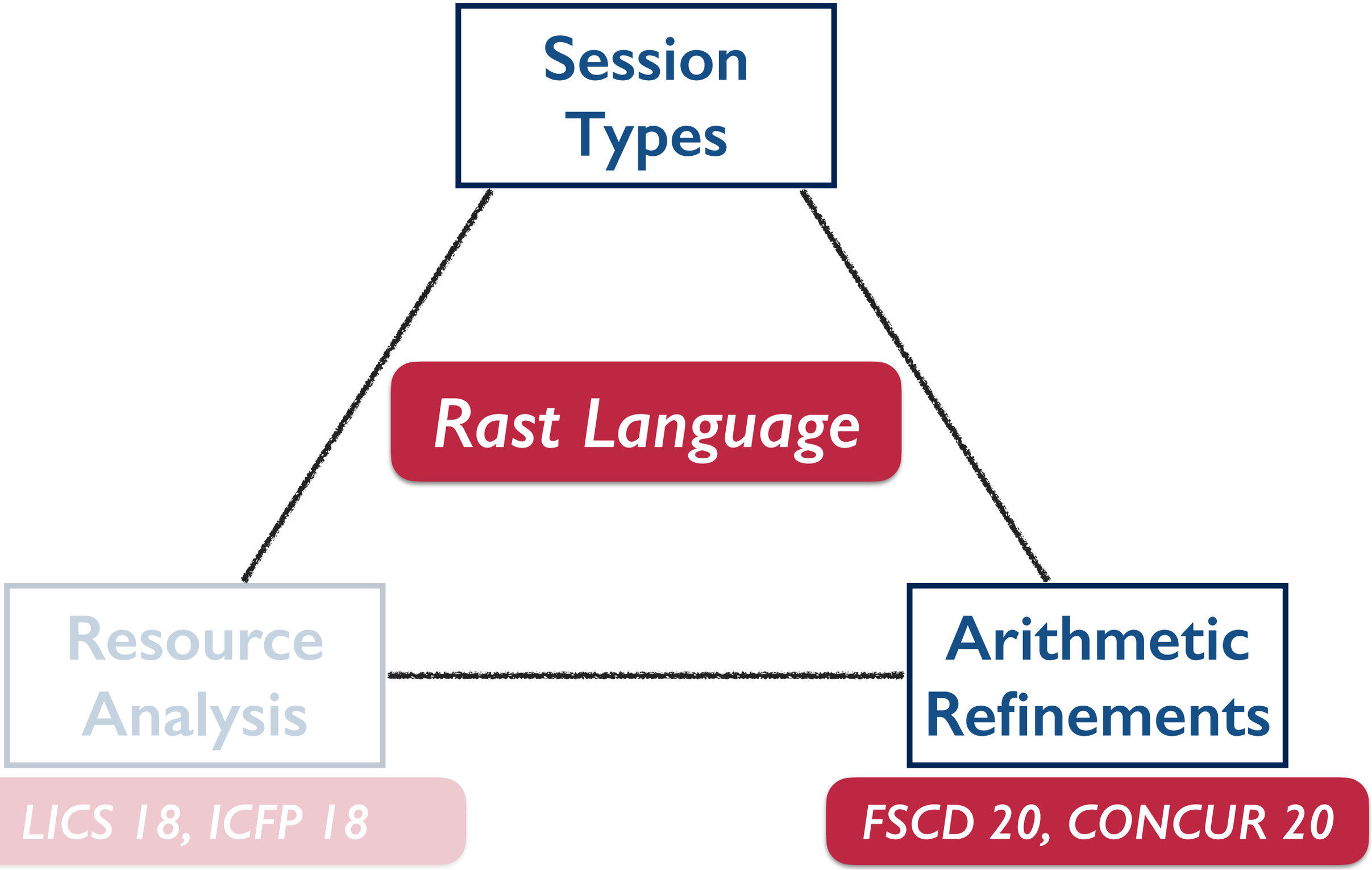

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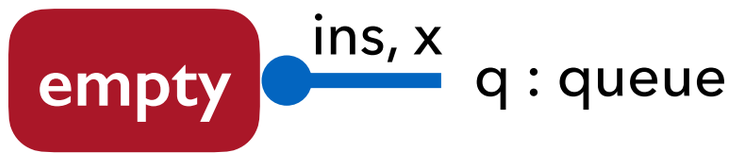

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decl elem{n | n > 0} : (x : A) (t : queue{n-1}) |- (q : queue{n})
```

```
proc q <- empty =
  case q (
    ins => x <- recv q ;
           t <- empty ;
           q <- elem{1} x t
  | del => q.none ;
           assert q {0 = 0} ;
           close q )
```

```
proc q <- elem{n} x t =
  case q (
    ins => y <- recv q ;
           t.ins ;
           send t y ;
           q <- elem{n+1} x t
  | del => q.some ;
           assert q {n > 0} ;
           send q x ;
           q <-> t )
```

send constraint

Key Observation

sending a proof corresponds to an *assertion*
receiving a proof corresponds to an *assumption*

Two New Constructs

assert x {ϕ}: send constraint ϕ along channel x

assume x {ϕ}: receive constraint ϕ along channel x

Formal Typing Judgment

$$\mathcal{V} \; ; \; C \; ; \; \Delta \vdash_{\Sigma} P :: (x : A)$$

Process P uses channels in Δ and offers channel $x : A$ under free variables $\mathcal{V}$ satisfying constraint C

Formal Typing Judgment

Free variables

$$\mathcal{V} \; ; \; C \; ; \; \Delta \vdash_{\Sigma} P :: (x : A)$$

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Formal Typing Judgment

Constraint satisfied by V

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Context (Channel Types)

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Process Expression

Process P uses channels in Δ and offers channel $x : A$ under free variables $\mathcal{V}$ satisfying constraint C

Formal Typing Judgment

Constraint satisfied by V

Free variables

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Process Expression

Offered Channel Type

Process P uses channels in Δ and offers channel $x : A$ under free variables $\mathcal{V}$ satisfying constraint C

Typing Rules

$$\frac{\mathcal{V} \;;\; C \vDash \phi \quad \mathcal{V} \;;\; C \;;\; \Delta \vdash P :: (x : A)}{\mathcal{V} \;;\; C \;;\; \Delta \vdash \mathsf{assert}\ x\ \{\phi\} \;;\; P :: (x : ?\{\phi\}.\, A)}\ ?R$$

$$\frac{\mathcal{V} \;;\; C \wedge \phi \;;\; \Delta, (x : A) \vdash Q :: (z : C)}{\mathcal{V} \;;\; C \;;\; \Delta, (x : ?\{\phi\}.\, A) \vdash \mathsf{assume}\ x\ \{\phi\} \;;\; Q :: (z : C)}\ ?L$$

Typing Rules

C proves ϕ

$$\frac{\mathcal{V}\ ;\ C \vDash \phi \quad \mathcal{V}\ ;\ C\ ;\ \Delta \vdash P :: (x : A)}{\mathcal{V}\ ;\ C\ ;\ \Delta \vdash \text{assert}\ x\ \{\phi\}\ ;\ P :: (x : ?\{\phi\}.\, A)}\ ?R$$

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Typing Rules

C proves ϕ

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Add ϕ to C

$$\frac{\mathcal{V}\ ;\ C \wedge \phi\ ;\ \Delta, (x : A) \vdash Q :: (z : C)}{\mathcal{V}\ ;\ C\ ;\ \Delta, (x : ?\{\phi\}.\, A) \vdash \text{assume}\ x\ \{\phi\}\ ;\ Q :: (z : C)}\ ?L$$

Too Many Asserts/Assumes!

$$\mathrm{nat}[n] = \oplus\{\mathbf{succ} : ?\{n > 0\}.\, \mathrm{nat}[n-1],\ \mathbf{zero} : ?\{n = 0\}.\, \mathbf{1}\}$$

$$\mathit{predecessor}[n] : (x : \mathrm{nat}[n+1]) \vdash (y : \mathrm{nat}[n])$$

$$y \leftarrow \mathit{predecessor}[n]\ x =$$
$$\quad \mathrm{case}\ x\ (\mathbf{succ} \Rightarrow y \leftrightarrow x)$$

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```
y ← predecessor[n] x =
    case x (
        succ ⇒ y ↔ x)
```

```
y ← predecessor[n] x =
    case x (
        succ ⇒ assume x {n + 1 > 0} ;
                y ↔ x
      | zero ⇒ assume x {n + 1 = 0} ;
                impossible)
```

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```

Insert asserts/assumes/impossible automatically?

Program Reconstruction

- **Key Idea : Treat constraints as *money* and the reconstruction engine as a *greedy salesperson***
- **Eagerly insert assumes (*get money*) and lazily insert asserts (*pay money*)**
- **Logically: type rules for assumes are invertible, therefore applied eagerly**
- **Formalized using the forcing calculus**

Forcing Calculus

$$\mathcal{V} \; ; \; \mathcal{C} \; ; \; \Delta \vdash_{\Sigma} P :: (x : A)$$

Forcing Calculus

$$\mathcal{V} \; ; \; C \; ; \; \Delta \vdash_{\Sigma} P :: (x : A)$$

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Forcing Calculus

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invertible rules applied

invertible rules not applied

- **Δ : linear context, corresponding assumes inserted**
- **Ω : ordered context, assumes to be inserted**

Reconstruction Properties

$$\mathit{If}\ \mathcal{V}\ ;\ C\ ;\ \cdot\ ;\ \Delta \vdash P :: (x : A),\ \mathit{then}\ \mathcal{V}\ ;\ C\ ;\ \Delta\ {}_{i}\!\vdash P :: (x : A).$$

Soundness

$$\mathit{If}\ \mathcal{V}\ ;\ C\ ;\ \Delta\ {}_{i}\!\vdash P :: (x : A),\ \mathit{then}\ \mathcal{V}\ ;\ C\ ;\ \cdot\ ;\ \Delta \vdash P :: (x : A).$$

Completeness

Reconstruction Properties

$$If\, \mathcal{V}\ ;\ C\ ;\ \cdot\ ;\ \Delta \vdash P :: (x : A),\ then\, \mathcal{V}\ ;\ C\ ;\ \Delta\ {}_i\!\vdash P :: (x : A).$$

Soundness

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Completeness

- **Soundness: the program that the reconstruction engine generates is well-typed!**
- **Completeness: if there is a reconstruction possible, our engine will find it!**

Examples in the Rast Programming Language

List Operations


```
type list{n} = +{cons : ?{n > 0}. A * list{n-1},
                 nil : ?{n = 0}. 1}
```

List Operations


```
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```

```
decl cons{n} : (x : A) (t : list{n}) |- (l : list{n+1})

decl append{n1}{n2} : (l1 : list{n1}) (l2 : list{n2}) |- (l : list{n1+n2})

decl split{n} : (l : list{2*n}) |- (m : list{n} * list{n})

decl reverse{n} : (l : list{n}) |- (m : list{n})

decl map{n} : (k : list{n}) (m : mapper) |- (l : list{n})

decl filter{n} : (s : selector) (k : list{n}) |- (l : ?m. ?{m <= n}. list{m})
```

Linear λ-Calculus


```
type exp = +{ lam : exp -o exp,
              app : exp * exp }

type val = +{ lam : exp -o exp }
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Linear λ-Calculus


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expression sizes

```
type exp{n} = +{lam : ?{n > 0}. !n1.exp{n1} -o exp{n1+n-1},
                app : ?n1. ?n2. ?{n = n1+n2+1}. exp{n1} * exp{n2}}

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type val{n} = +{lam : ?{n > 0}. !n1.exp{n1} -o exp{n1+n-1}}

decl eval{n} : (e : exp{n}) |- (v : ?k. ?{k <= n}. val{k})
```

size of value is smaller

Evaluation

Module	iLOC	eLOC	R (ms)
arithmetic	69	143	0.353
integers	90	114	0.200
linlam	54	67	0.734
list	244	441	1.534
primes	90	118	0.196
segments	48	65	0.239
ternary	156	235	0.550
theorems	79	141	0.361
tries	147	308	1.113
Total	**977**	**1632**	**5.280**

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67% increase in lines of code after recon!

Evaluation

Before Reconstruction

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67% increase in lines of code after recon!

recon is very efficient! <2ms

The Rast Language

- *Resource-Aware Session Types:* refinement session types with support for verifying *sequential* and *parallel* complexity bounds automatically
- *Lightweight verification* using refinements
- *Reconstruction:* constructs pertaining to refinement layer are inserted automatically
- *Evaluation:* implemented standard benchmarks
- *Availability:* implementation open-source on https://bitbucket.org/fpfenning/rast/src/master/rast/