

Arg

View the online version at
● functions.wolfram.com

Download the
● PDF File

Notations

Traditional name

Argument

Traditional notation

$\arg(z)$

Mathematica StandardForm notation

$\text{Arg}[z]$

Primary definition

12.02.02.0001.01

$$\arg(z) = -i \log\left(\frac{z}{|z|}\right)$$

$\text{Arg}(z)$ is the argument of z , such that $z = |z| e^{i \text{Arg}(z)}$. The argument of a complex number z is the phase angle (in radians) that the line from 0 to z makes with the positive real axis.

Specific values

Specialized values

12.02.03.0001.01

$$\arg(x) = 0 \text{ ; } x \in \mathbb{R} \wedge x > 0$$

12.02.03.0020.01

$$\arg(x) = \pi \text{ ; } x \in \mathbb{R} \wedge x < 0$$

12.02.03.0002.01

$$\arg(ix) = \frac{\pi}{2} \text{ ; } x \in \mathbb{R} \wedge x > 0$$

12.02.03.0021.01

$$\arg(ix) = -\frac{\pi}{2} \text{ ; } x \in \mathbb{R} \wedge x < 0$$

12.02.03.0003.01

$$\arg(x + iy) = \tan^{-1}(x, y) \text{ ; } x \in \mathbb{R} \wedge y \in \mathbb{R}$$

Values at fixed points

12.02.03.0004.01

$$\arg(0) \in (-\pi, \pi]$$

$\text{Arg}(0)$ is not a uniquely defined number. Depending on the argument of z , the limit $\lim_{|z| \rightarrow 0} \text{Arg}(z)$ can take any value in the interval $(-\pi, \pi)$.

12.02.03.0005.01

$$\arg(1) = 0$$

12.02.03.0006.01

$$\arg(-1) = \pi$$

12.02.03.0007.01

$$\arg(i) = \frac{\pi}{2}$$

12.02.03.0008.01

$$\arg(-i) = -\frac{\pi}{2}$$

12.02.03.0022.01

$$\arg(1 + i) = \frac{\pi}{4}$$

12.02.03.0023.01

$$\arg(-1 + i) = \frac{3\pi}{4}$$

12.02.03.0024.01

$$\arg(-1 - i) = -\frac{3\pi}{4}$$

12.02.03.0025.01

$$\arg(1 - i) = -\frac{\pi}{4}$$

12.02.03.0026.01

$$\arg(\sqrt{3} + i) = \frac{\pi}{6}$$

12.02.03.0027.01

$$\arg(1 + i\sqrt{3}) = \frac{\pi}{3}$$

12.02.03.0028.01

$$\arg(-1 + i\sqrt{3}) = \frac{2\pi}{3}$$

12.02.03.0029.01

$$\arg(-\sqrt{3} + i) = \frac{5\pi}{6}$$

12.02.03.0030.01

$$\arg(-\sqrt{3} - i) = -\frac{5\pi}{6}$$

12.02.03.0031.01

$$\arg(-1 - i\sqrt{3}) = -\frac{2\pi}{3}$$

12.02.03.0032.01

$$\arg(1 - i\sqrt{3}) = -\frac{\pi}{3}$$

12.02.03.0033.01

$$\arg(\sqrt{3} - i) = -\frac{\pi}{6}$$

12.02.03.0009.01

$$\arg(2) = 0$$

12.02.03.0010.01

$$\arg(-2) = \pi$$

12.02.03.0011.01

$$\arg(\pi) = 0$$

12.02.03.0012.01

$$\arg(3i) = \frac{\pi}{2}$$

12.02.03.0013.01

$$\arg(-2i) = -\frac{\pi}{2}$$

12.02.03.0014.01

$$\arg(2 + i) = \tan^{-1}\left(\frac{1}{2}\right)$$

Values at infinities

12.02.03.0015.01

$$\arg(\infty) = 0$$

12.02.03.0016.01

$$\arg(-\infty) = \pi$$

12.02.03.0017.01

$$\arg(i\infty) = \frac{\pi}{2}$$

12.02.03.0018.01

$$\arg(-i\infty) = -\frac{\pi}{2}$$

12.02.03.0019.01

$$\arg(\infty) \in (-\pi, \pi]$$

General characteristics

Domain and analyticity

$\text{Arg}(z)$ is a nonanalytical function; it is a real-analytic function of the complex variable z for $z \neq 0$.

12.02.04.0001.01

$$z \longrightarrow \arg(z) :: \mathbb{C} \longrightarrow \mathbb{R}$$

Symmetries and periodicities

Parity

$\text{Arg}(z)$ is an odd function for almost all z .

12.02.04.0002.01

$$\arg(-z) = -\arg(z) \text{ ; } z \notin (-\infty, 0)$$

12.02.04.0003.01

$$\arg(-z) = \arg(z) - \frac{\sqrt{-z}}{\sqrt{z}} i \pi$$

Mirror symmetry

12.02.04.0004.01

$$\arg(\bar{z}) = -\arg(z) \text{ ; } z \notin (-\infty, 0)$$

Periodicity

No periodicity

Homogeneity

12.02.04.0005.01

$$\arg(az) = \arg(z) \text{ ; } a \in \mathbb{R} \wedge a > 0$$

Sets of discontinuity

The function $\text{Arg}(z)$ is a single-valued, continuous function on the z -plane cut along the interval $(-\infty, 0)$, where it is continuous from above.

12.02.04.0006.01

$$\mathcal{DS}_z(\arg(z)) = \{(-\infty, 0), -i\}$$

12.02.04.0007.01

$$\lim_{\epsilon \rightarrow +0} \arg(x + i\epsilon) = \arg(x) = \pi \text{ ; } x \in \mathbb{R} \wedge x < 0$$

12.02.04.0008.01

$$\lim_{\epsilon \rightarrow +0} \arg(x - i\epsilon) = -\pi \text{ ; } x \in \mathbb{R} \wedge x < 0$$

Transformations

Transformations and argument simplifications

Argument involving complex characteristics

12.02.16.0032.01

$$\arg(|z|) = 0$$

12.02.16.0033.01

$$\arg\left(\frac{z}{|z|}\right) = \arg(z)$$

12.02.16.0034.01

$$\arg(\operatorname{sgn}(z)) = \arg(z)$$

12.02.16.0006.01

$$\arg(\bar{z}) = -\arg(z) \text{ ; } \arg(z) \neq \pi$$

12.02.16.0035.01

$$\arg(\bar{z}) = 2\pi \left\lfloor \frac{\arg(z) + \pi}{2\pi} \right\rfloor - \arg(z)$$

Argument involving basic arithmetic operations

12.02.16.0001.01

$$\arg(-z) = -\arg(z) \text{ ; } z \notin (-\infty, 0)$$

12.02.16.0002.01

$$\arg(-z) = \arg(z) - \frac{\sqrt{-z}}{\sqrt{z}} i \pi$$

12.02.16.0036.01

$$\arg(-z) = \arg(z) + \pi \left(2 \left\lfloor -\frac{\arg(z)}{2\pi} \right\rfloor + 1 \right)$$

12.02.16.0037.01

$$\arg(i z) = \arg(z) + \frac{\pi}{2} \text{ ; } \arg(z) \leq \frac{\pi}{2}$$

12.02.16.0038.01

$$\arg(i z) = \arg(z) - \frac{3\pi}{2} \text{ ; } \arg(z) > \frac{\pi}{2}$$

12.02.16.0004.01

$$\arg(i z) = \arg(z) - \frac{\pi}{2} - \frac{(-1)^{3/4} \pi \sqrt{i z}}{\sqrt{z}}$$

12.02.16.0039.01

$$\arg(i z) = \arg(z) + 2\pi \left\lfloor \frac{1}{4} - \frac{\arg(z)}{2\pi} \right\rfloor + \frac{\pi}{2}$$

12.02.16.0040.01

$$\arg(-i z) = \arg(z) - \frac{\pi}{2} \text{ ; } \arg(z) > -\frac{\pi}{2}$$

12.02.16.0041.01

$$\arg(-i z) = \arg(z) + \frac{3\pi}{2} \text{ ; } \arg(z) \leq -\frac{\pi}{2}$$

12.02.16.0005.01

$$\arg(-i z) = \arg(z) + \frac{\pi}{2} - \frac{\sqrt[4]{-1} \pi \sqrt{-i z}}{\sqrt{z}}$$

12.02.16.0042.01

$$\arg(-i z) = \arg(z) + 2\pi \left\lfloor \frac{3}{4} - \frac{\arg(z)}{2\pi} \right\rfloor - \frac{\pi}{2}$$

12.02.16.0007.01

$$\arg\left(\frac{1}{z}\right) = -\arg(z) \text{ ; } z \notin (-\infty, 0)$$

12.02.16.0008.01

$$\arg\left(\frac{1}{z}\right) = -\arg(z) \text{ ; } \arg(z) \neq \pi$$

12.02.16.0043.01

$$\arg\left(\frac{1}{z}\right) = 2\pi - \arg(z) \text{ ; } \arg(z) = \pi$$

12.02.16.0009.01

$$\arg\left(\frac{1}{z}\right) = -\sqrt{z} \sqrt{\frac{1}{z}} \arg(z)$$

12.02.16.0044.01

$$\arg\left(\frac{1}{z}\right) = 2\pi \left\lfloor \frac{\arg(z) + \pi}{2\pi} \right\rfloor - \arg(z)$$

12.02.16.0045.01

$$\arg\left(-\frac{1}{z}\right) = \pi - \arg(z) \text{ ; } \operatorname{Im}(z) \geq 0$$

12.02.16.0046.01

$$\arg\left(-\frac{1}{z}\right) = -\arg(z) - \pi \text{ ; } \operatorname{Im}(z) < 0$$

12.02.16.0047.01

$$\arg\left(-\frac{1}{z}\right) = -\arg(z) - \pi i \sqrt{-\frac{1}{z}} \sqrt{z}$$

12.02.16.0048.01

$$\arg\left(-\frac{1}{z}\right) = -\arg(z) + 2\pi \left\lfloor \frac{\arg(z)}{2\pi} \right\rfloor + \pi$$

12.02.16.0049.01

$$\arg\left(\frac{i}{z}\right) = \frac{\pi}{2} - \arg(z) \text{ ; } \arg(z) \geq -\frac{\pi}{2}$$

12.02.16.0050.01

$$\arg\left(\frac{i}{z}\right) = -\arg(z) - \frac{3\pi}{2} \text{ ; } \arg(z) < -\frac{\pi}{2}$$

12.02.16.0051.01

$$\arg\left(\frac{i}{z}\right) = -\arg(z) + 2\pi \left\lfloor \frac{\arg(z)}{2\pi} + \frac{1}{4} \right\rfloor + \frac{\pi}{2}$$

12.02.16.0052.01

$$\arg\left(-\frac{i}{z}\right) = -\frac{\pi i}{2} - \arg(z) \text{ ; } \arg(z) < \frac{\pi}{2}$$

12.02.16.0053.01

$$\arg\left(-\frac{i}{z}\right) = \frac{3\pi}{2} - \arg(z) \text{ ; } \arg(z) \geq \frac{\pi}{2}$$

12.02.16.0054.01

$$\arg\left(-\frac{i}{z}\right) = -\arg(z) + 2\pi \left\lfloor \frac{\arg(z)}{2\pi} + \frac{3}{4} \right\rfloor - \frac{\pi}{2}$$

Addition formulas

12.02.16.0010.01

$$\arg(x + iy) = \tan^{-1}(x, y) \text{ ; } \operatorname{Im}(x) = 0 \wedge \operatorname{Im}(y) = 0$$

Multiple arguments

For products

12.02.16.0011.01

$$\arg(az) = \arg(z) \text{ ; } a \in \mathbb{R} \wedge a > 0$$

12.02.16.0014.01

$$\arg(z_1 z_2) = \arg(z_1) + \arg(z_2) \text{ ; } -\pi < \arg(z_1) + \arg(z_2) \leq \pi$$

12.02.16.0055.01

$$\arg(z - z^2) = \arg(1 - z) + \arg(z)$$

12.02.16.0056.01

$$\arg(-z - z^2) = \arg(1 + z) + \arg(-z)$$

12.02.16.0057.01

$$\arg(z_1 z_2) = \arg(z_1) + \arg(z_2) \text{ ; } \arg(z_1) \leq 0 \wedge -\arg(z_1) - \pi < \arg(z_2) \vee \arg(z_1) \geq 0 \wedge \arg(z_2) \leq \pi - \arg(z_1)$$

12.02.16.0058.01

$$\arg(z_1 z_2) = \arg(z_1) + \arg(z_2) - 2\pi \text{ ; } \arg(z_1) \geq 0 \wedge \arg(z_2) > \pi - \arg(z_1)$$

12.02.16.0059.01

$$\arg(z_1 z_2) = \arg(z_1) + \arg(z_2) + 2\pi \text{ ; } \arg(z_1) \leq 0 \wedge \arg(z_2) \leq -\arg(z_1) - \pi$$

12.02.16.0015.01

$$\arg(z_1 z_2) = \arg(z_1) + \arg(z_2) + 2\pi \left\lfloor \frac{\pi - \arg(z_1) - \arg(z_2)}{2\pi} \right\rfloor$$

12.02.16.0060.01

$$\arg\left(\prod_{k=1}^n z_k\right) = \sum_{k=1}^n \arg(z_k) + 2\pi \left\lfloor \frac{\pi - \sum_{k=1}^n \arg(z_k)}{2\pi} \right\rfloor \text{ ; } n \in \mathbb{N}^+$$

For quotients

12.02.16.0061.01

$$\arg\left(\frac{z}{z+1}\right) = \arg(z) - \arg(z+1)$$

12.02.16.0062.01

$$\arg\left(\frac{z}{z-1}\right) = \arg(-z) - \arg(1-z)$$

12.02.16.0016.01

$$\arg\left(\frac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2) \text{ ; } -\pi < \arg(z_1) - \arg(z_2) \leq \pi$$

12.02.16.0063.01

$$\arg\left(\frac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2) \text{ ; } \arg(z_1) \leq 0 \wedge \arg(z_2) < \arg(z_1) + \pi \vee \arg(z_1) > 0 \wedge \arg(z_2) \geq \arg(z_1) - \pi$$

12.02.16.0064.01

$$\arg\left(\frac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2) - 2\pi \text{ ; } \arg(z_1) \geq 0 \wedge \arg(z_2) < \arg(z_1) - \pi$$

12.02.16.0065.01

$$\arg\left(\frac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2) + 2\pi \text{ ; } \arg(z_1) \leq 0 \wedge \arg(z_2) \geq \arg(z_1) + \pi$$

12.02.16.0017.01

$$\arg\left(\frac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2) + 2\pi \left\lfloor \frac{\pi - \arg(z_1) + \arg(z_2)}{2\pi} \right\rfloor$$

Power of arguments

12.02.16.0066.01

$$\arg(\sqrt{z}) = \frac{\arg(z)}{2}$$

12.02.16.0067.01

$$\arg(\sqrt{z}) = \arg(z) \text{ ; } \operatorname{Re}(z) > 0 \vee \operatorname{Re}(z) = 0 \wedge \operatorname{Im}(z) > 0$$

12.02.16.0068.01

$$\arg(\sqrt{z^2}) = \arg(z) - \frac{\pi i (\sqrt{z^2} - z)}{2 \sqrt{-z^2}}$$

12.02.16.0069.01

$$\arg(z^{1/n}) = \frac{\arg(z)}{n} \text{ ; } n \in \mathbb{Z} \wedge n \neq 0 \wedge n \neq -1$$

12.02.16.0070.01

$$\arg(z^2) = 2 \arg(z) \text{ ; } \operatorname{Re}(z) > 0 \vee \operatorname{Re}(z) = 0 \wedge \operatorname{Im}(z) > 0$$

12.02.16.0071.01

$$\arg(z^2) = 2 \arg(z) + 2\pi \text{ ; } -\pi < \arg(z) \leq -\frac{\pi}{2}$$

12.02.16.0072.01

$$\arg(z^2) = 2 \arg(z) - 2\pi \text{ ; } \frac{\pi}{2} < \arg(z) \leq \pi$$

12.02.16.0073.01

$$\arg(z^2) = 2 \arg(z) + 2\pi \left\lfloor \frac{1}{2} - \frac{\arg(z)}{\pi} \right\rfloor$$

12.02.16.0074.01

$$\arg(z^2) = 2 \arg(z) - \frac{\pi i \left(\sqrt{z^2} - z \right)}{\sqrt{-z^2}}$$

12.02.16.0018.01

$$\arg(x^a) = \tan^{-1}(\cos(\operatorname{Im}(a) \log(x)), \sin(\operatorname{Im}(a) \log(x))) /; x \in \mathbb{R} \wedge x > 0$$

12.02.16.0019.01

$$\arg(z^a) = a \arg(z) /; a \in \mathbb{R} \wedge -\pi < a \arg(z) \leq \pi$$

12.02.16.0075.01

$$\arg(z^a) = a \arg(z) + 2 \pi k /; a \in \mathbb{R} \wedge -\pi - 2 \pi k < a \arg(z) \leq \pi - 2 \pi k \wedge k \in \mathbb{Z}$$

12.02.16.0076.01

$$\arg(z^a) = \operatorname{Im}(a \log(z)) /; -\pi < \operatorname{Im}(a \log(z)) \leq \pi$$

12.02.16.0077.01

$$\arg(z^a) = \operatorname{Im}(a \log(z)) + 2 \pi k /; -2 \pi k - \pi < \operatorname{Im}(a \log(z)) \leq \pi - 2 \pi k \wedge k \in \mathbb{Z}$$

12.02.16.0020.01

$$\arg(z^a) = \arg(e^{i a \arg(z)}) /; a \in \mathbb{R}$$

12.02.16.0021.01

$$\arg(z^a) = \tan^{-1}(\cos(a \tan^{-1}(\operatorname{Re}(z), \operatorname{Im}(z))), \sin(a \tan^{-1}(\operatorname{Re}(z), \operatorname{Im}(z)))) /; a \in \mathbb{R}$$

12.02.16.0022.01

$$\arg(x^a) = \operatorname{Im}(a) \log(x) /; -\frac{\pi}{\log(x)} < \operatorname{Im}(a) \leq \frac{\pi}{\log(x)} \wedge x \in \mathbb{R} \wedge x > 0$$

12.02.16.0024.01

$$\arg(z^a) = a \arg(z) + 2 \pi \left\lfloor \frac{\pi - a \arg(z)}{2 \pi} \right\rfloor /; \operatorname{Im}(a) = 0$$

12.02.16.0078.01

$$\arg(z^a) = a \arg(z) + \operatorname{Im}(a) \overline{\log(z)} + 2 \pi \left\lfloor \frac{\pi - \operatorname{Im}(a \log(z))}{2 \pi} \right\rfloor$$

12.02.16.0079.01

$$\arg(z^a) = a \arg(z) + 2 \pi \left\lfloor \frac{\pi - \operatorname{Im}(a \log(z))}{2 \pi} \right\rfloor - i a \log(|z|) + i \operatorname{Re}(a \log(z))$$

12.02.16.0025.01

$$\arg(z^a) = 2 \pi \left\lfloor \frac{\pi - \operatorname{Im}(a \log(z))}{2 \pi} \right\rfloor + \operatorname{Im}(a \log(z))$$

12.02.16.0080.01

$$\arg(z^a) = \operatorname{Im}(a) \log(|z|) + \arg(z) \operatorname{Re}(a) + 2 \pi \left\lfloor \frac{\pi - \operatorname{Im}(a \log(z))}{2 \pi} \right\rfloor$$

12.02.16.0026.01

$$\arg(z^a) = \operatorname{Re}(a) \arg(z) + \operatorname{Im}(a) \log(|z|) + 2 \pi \left\lfloor \frac{\pi - \operatorname{Im}(a) \log(|z|) - \arg(z) \operatorname{Re}(a)}{2 \pi} \right\rfloor$$

12.02.16.0027.01

$$\arg(z^a) = \tan^{-1}(\cos(\operatorname{Im}(a) \log(|z|) + \tan^{-1}(\operatorname{Re}(z), \operatorname{Im}(z)) \operatorname{Re}(a)), \sin(\operatorname{Im}(a) \log(|z|) + \tan^{-1}(\operatorname{Re}(z), \operatorname{Im}(z)) \operatorname{Re}(a)))$$

Exponent of arguments

12.02.16.0081.01

$$\arg(e^{x+iy}) = \tan^{-1}(\cos(y), \sin(y))$$

12.02.16.0082.01

$$\arg(e^z) = \operatorname{Im}(z) \text{ ; } -\pi < \operatorname{Im}(z) \leq \pi$$

12.02.16.0083.01

$$\arg(e^z) = 2\pi k + \operatorname{Im}(z) \text{ ; } -2\pi k - \pi < \operatorname{Im}(z) \leq \pi - 2\pi k \wedge k \in \mathbb{Z}$$

12.02.16.0084.01

$$\arg(e^z) = \operatorname{Im}(z) + 2\pi \left\lfloor \frac{\pi - \operatorname{Im}(z)}{2\pi} \right\rfloor$$

12.02.16.0085.01

$$\arg(e^{iz}) = \operatorname{Re}(z) + 2\pi \left\lfloor \frac{\pi - \operatorname{Re}(z)}{2\pi} \right\rfloor$$

12.02.16.0086.01

$$\arg(e^z) = \pi - (\pi - \operatorname{Im}(z)) \bmod (2\pi)$$

12.02.16.0087.01

$$\arg(e^{iz}) = \pi - (\pi - \operatorname{Re}(z)) \bmod (2\pi)$$

Some functions of arguments

12.02.16.0088.01

$$\arg(c z^a) = \arg(c) + \operatorname{Im}(a \log(z)) + 2\pi \left\lfloor \frac{\pi - \arg(c) - \operatorname{Im}(a \log(z))}{2\pi} \right\rfloor$$

12.02.16.0089.01

$$\arg(c e^z) = \arg(c) + \operatorname{Im}(z) + 2\pi \left\lfloor \frac{\pi - \arg(c) - \operatorname{Im}(z)}{2\pi} \right\rfloor$$

12.02.16.0090.01

$$\arg(x^a y^b) = a \arg(x) + b \arg(y) + 2\pi \left\lfloor \frac{-a \arg(x) - b \arg(y) + \pi}{2\pi} \right\rfloor \text{ ; } a \in \mathbb{R} \wedge b \in \mathbb{R}$$

12.02.16.0091.01

$$\arg(x^a y^b z^c) = a \arg(x) + b \arg(y) + c \arg(z) + 2\pi \left\lfloor \frac{-a \arg(x) - b \arg(y) - c \arg(z) + \pi}{2\pi} \right\rfloor \text{ ; } a \in \mathbb{R} \wedge b \in \mathbb{R} \wedge c \in \mathbb{R}$$

12.02.16.0092.01

$$\arg\left(\prod_{k=1}^n z_k^{a_k}\right) = \sum_{k=1}^n a_k \arg(z_k) + 2\pi \left\lfloor \frac{\pi - \sum_{k=1}^n a_k \arg(z_k)}{2\pi} \right\rfloor \text{ ; } a_k \in \mathbb{R} \wedge 1 \leq k \leq n$$

12.02.16.0093.01

$$\arg(x^a y^b) = 2\pi \left\lfloor \frac{\pi - \operatorname{Im}(a \log(x)) - \operatorname{Im}(b \log(y))}{2\pi} \right\rfloor + \operatorname{Im}(a \log(x)) + \operatorname{Im}(b \log(y))$$

12.02.16.0094.01

$$\arg(x^a y^b z^c) = 2\pi \left\lfloor \frac{\pi - \operatorname{Im}(a \log(x)) - \operatorname{Im}(b \log(y)) - \operatorname{Im}(c \log(z))}{2\pi} \right\rfloor + \operatorname{Im}(a \log(x)) + \operatorname{Im}(b \log(y)) + \operatorname{Im}(c \log(z))$$

$$\arg\left(\prod_{k=1}^n z_k^{a_k}\right) = 2\pi \left\lfloor \frac{\pi - \sum_{k=1}^n \operatorname{Im}(a_k \log(z_k))}{2\pi} \right\rfloor + \sum_{k=1}^n \operatorname{Im}(a_k \log(z_k))$$

Products, sums, and powers of the direct function

Sums of the direct function

$$\arg(z_1) + \arg(z_2) = \arg(z_1 z_2) \text{ ; } \arg(z_1) \leq 0 \wedge -\arg(z_1) - \pi < \arg(z_2) \vee \arg(z_1) \geq 0 \wedge \arg(z_2) \leq \pi - \arg(z_1)$$

$$\arg(z_1) + \arg(z_2) = \arg(z_1 z_2) + 2\pi \text{ ; } \arg(z_1) \geq 0 \wedge \arg(z_2) > \pi - \arg(z_1)$$

$$\arg(z_1) + \arg(z_2) = \arg(z_1 z_2) - 2\pi \text{ ; } \arg(z_1) \leq 0 \wedge \arg(z_2) \leq -\arg(z_1) - \pi$$

$$\arg(z_1) + \arg(z_2) = \arg(z_1 z_2) - 2\pi \left\lfloor \frac{\pi - \arg(z_1) - \arg(z_2)}{2\pi} \right\rfloor$$

$$\sum_{k=1}^n \arg(z_k) = \arg\left(\prod_{k=1}^n z_k\right) - 2\pi \left\lfloor \frac{\pi - \sum_{k=1}^n \arg(z_k)}{2\pi} \right\rfloor \text{ ; } n \in \mathbb{N}^+$$

Differences of the direct function

$$\arg(z_1) - \arg(z_2) = \arg\left(\frac{z_1}{z_2}\right) \text{ ; } -\pi < \arg(z_1) - \arg(z_2) \leq \pi$$

$$\arg(z_1) - \arg(z_2) = \arg\left(\frac{z_1}{z_2}\right) \text{ ; } \arg(z_1) \leq 0 \wedge \arg(z_2) < \arg(z_1) + \pi \vee \arg(z_1) > 0 \wedge \arg(z_2) \geq \arg(z_1) - \pi$$

$$\arg(z_1) - \arg(z_2) = \arg\left(\frac{z_1}{z_2}\right) + 2\pi \text{ ; } \arg(z_1) \geq 0 \wedge \arg(z_2) < \arg(z_1) - \pi$$

$$\arg(z_1) - \arg(z_2) = \arg\left(\frac{z_1}{z_2}\right) - 2\pi \text{ ; } \arg(z_1) \leq 0 \wedge \arg(z_2) \geq \arg(z_1) + \pi$$

$$\arg(z_1) - \arg(z_2) = \arg\left(\frac{z_1}{z_2}\right) - 2\pi \left\lfloor \frac{\arg(z_2) - \arg(z_1) + \pi}{2\pi} \right\rfloor$$

Linear combinations of the direct function

$$a \arg(x) + b \arg(y) = \arg(x^a y^b) - 2\pi \left\lfloor \frac{\pi - a \arg(x) - b \arg(y)}{2\pi} \right\rfloor \text{ ; } a \in \mathbb{R} \wedge b \in \mathbb{R}$$

$$a \arg(x) + b \arg(y) + c \arg(z) = \arg(x^a y^b z^c) - 2\pi \left\lfloor \frac{\pi - a \arg(x) - b \arg(y) - c \arg(z)}{2\pi} \right\rfloor \text{ ; } a \in \mathbb{R} \wedge b \in \mathbb{R} \wedge c \in \mathbb{R}$$

12.02.16.0107.01

$$\sum_{k=1}^n a_k \arg(z_k) = \arg\left(\prod_{k=1}^n z_k^{a_k}\right) - 2\pi \left\lfloor \frac{\pi - \sum_{k=1}^n a_k \arg(z_k)}{2\pi} \right\rfloor; a_k \in \mathbb{R} \wedge 1 \leq k \leq n$$

Related transformations

12.02.16.0029.01

$$e^{i \arg(z)} = \frac{z}{|z|}$$

12.02.16.0030.01

$$e^{i \arg(z)} = \cos(\arg(z)) + i \sin(\arg(z))$$

12.02.16.0031.01

$$e^{i \arg(z)} = \cos\left(\tan^{-1}\left(\frac{\operatorname{Im}(z)}{\operatorname{Re}(z)}\right)\right) + i \sin\left(\tan^{-1}\left(\frac{\operatorname{Im}(z)}{\operatorname{Re}(z)}\right)\right); -\frac{\pi}{2} < \arg(\operatorname{Re}(z)) \leq \frac{\pi}{2}$$

Complex characteristics

Real part

12.02.19.0001.01

$$\operatorname{Re}(\arg(x + i y)) = \tan^{-1}(x, y)$$

12.02.19.0008.01

$$\operatorname{Re}(\arg(z)) = \arg(z)$$

Imaginary part

12.02.19.0002.01

$$\operatorname{Im}(\arg(x + i y)) = 0$$

12.02.19.0003.01

$$\operatorname{Im}(\arg(z)) = 0$$

Absolute value

12.02.19.0004.01

$$|\arg(x + i y)| = \sqrt{\tan^{-1}(x, y)^2}$$

12.02.19.0009.01

$$|\arg(z)| = \sqrt{\tan^{-1}(\operatorname{Re}(z), \operatorname{Im}(z))^2}$$

Argument

12.02.19.0005.01

$$\arg(\arg(x + i y)) = \tan^{-1}(\tan^{-1}(x, y), 0)$$

12.02.19.0010.01

$$\arg(\arg(z)) = \tan^{-1}(\tan^{-1}(\operatorname{Re}(z), \operatorname{Im}(z)), 0)$$

Conjugate value

12.02.19.0006.01

$$\overline{\arg(x + i y)} = \tan^{-1}(x, y)$$

12.02.19.0007.01

$$\overline{\arg(z)} = \arg(z)$$

Signum value

12.02.19.0011.01

$$\operatorname{sgn}(\arg(x + i y)) = \operatorname{sgn}(\tan^{-1}(x, y))$$

12.02.19.0012.01

$$\operatorname{sgn}(\arg(x + i y)) = \frac{\tan^{-1}(x, y)}{\sqrt{\tan^{-1}(x, y)^2}}$$

12.02.19.0013.01

$$\operatorname{sgn}(\arg(z)) = \operatorname{sgn}(\tan^{-1}(\operatorname{Re}(z), \operatorname{Im}(z)))$$

12.02.19.0014.01

$$\operatorname{sgn}(\arg(z)) = \frac{\arg(z)}{\sqrt{\tan^{-1}(\operatorname{Re}(z), \operatorname{Im}(z))^2}}$$

Representations through equivalent functions

With related functions

With Re

12.02.27.0006.01

$$\arg(z) = \tan^{-1}(\operatorname{Re}(z), -i(z - \operatorname{Re}(z)))$$

With Im

12.02.27.0007.01

$$\arg(z) = \tan^{-1}(z - i \operatorname{Im}(z), \operatorname{Im}(z))$$

12.02.27.0004.01

$$\arg(z) = \tan^{-1}(\operatorname{Re}(z), \operatorname{Im}(z))$$

12.02.27.0005.01

$$\arg(z) = \tan^{-1}\left(\frac{\operatorname{Im}(z)}{\operatorname{Re}(z)}\right); \operatorname{Re}(z) > 0$$

With Abs

12.02.27.0001.01

$$\arg(z) = -i \log\left(\frac{z}{|z|}\right)$$

12.02.27.0008.01

$$\arg(z) = i (\log(|z|) - \log(z))$$

12.02.27.0002.01

$$\cos(\arg(z)) = \frac{\operatorname{Re}(z)}{|z|}$$

12.02.27.0003.01

$$\sin(\arg(z)) = \frac{\operatorname{Im}(z)}{|z|}$$

With Conjugate

12.02.27.0009.01

$$\arg(z) = \frac{1}{2} i (\log(z \bar{z}) - 2 \log(z))$$

With Sign

12.02.27.0010.01

$$\arg(z) = -i \log(\operatorname{sgn}(z))$$

With inverse trigonometric functions**With ArcSin**

12.02.27.0011.01

$$\arg(z) = \sin^{-1}\left(\frac{\operatorname{Im}(z)}{|z|}\right) + \frac{\pi}{4} \left(-\sqrt{\frac{1}{z}} \sqrt{z} + \frac{i \sqrt{-z^2}}{z} \left(1 - \frac{\sqrt{z^2}}{z} \right) - \sqrt{\frac{i}{z}} \sqrt{-iz} + 2 \right)$$

12.02.27.0012.01

$$\arg(z) = \sin^{-1}\left(\frac{\operatorname{Re}(z)}{|z|}\right) + \frac{\pi}{4} \left(\frac{\sqrt{z^2}}{z} \left(1 + \frac{i \sqrt{-z^2}}{z} \right) - \sqrt{\frac{1}{z}} \sqrt{z} - \sqrt{-\frac{i}{z}} \sqrt{iz} + \frac{4(-1)^{3/4} \sqrt{iz}}{\sqrt{z}} + 4 \right)$$

With ArcCos

12.02.27.0013.01

$$\arg(z) = -\cos^{-1}\left(\frac{\operatorname{Im}(z)}{|z|}\right) + \frac{\pi}{4} \left(-\sqrt{\frac{1}{z}} \sqrt{z} + \frac{i \sqrt{-z^2}}{z} \left(1 - \frac{\sqrt{z^2}}{z} \right) - \sqrt{\frac{i}{z}} \sqrt{-iz} + 4 \right)$$

12.02.27.0014.01

$$\arg(z) = -\cos^{-1}\left(\frac{\operatorname{Re}(z)}{|z|}\right) + \frac{\pi}{4} \left(\frac{\sqrt{z^2}}{z} \left(1 + \frac{i \sqrt{-z^2}}{z} \right) - \sqrt{\frac{1}{z}} \sqrt{z} - \sqrt{-\frac{i}{z}} \sqrt{iz} + \frac{4(-1)^{3/4} \sqrt{iz}}{\sqrt{z}} + 6 \right)$$

With ArcTan

12.02.27.0015.01

$$\arg(z) = \tan^{-1}(\operatorname{Re}(z), \operatorname{Im}(z))$$

With inverse hyperbolic functions**Inequalities**

12.02.29.0001.01

$$|\arg(z)| \leq \pi$$

12.02.29.0002.01

$$-\pi < \arg(z) \leq \pi$$

12.02.29.0003.01

$$-r < \arg(a+z) < R /; -r < \arg(z) < R \wedge -r < \arg(a) < R \wedge R-r \leq \pi$$

Pavlyk O. (2006)

Zeros

12.02.30.0001.01

$$\arg(z) = 0 /; z \in \mathbb{R} \wedge z > 0$$

History

Arg is encountered often in mathematics and the natural sciences.

Copyright

This document was downloaded from functions.wolfram.com, a comprehensive online compendium of formulas involving the special functions of mathematics. For a key to the notations used here, see <http://functions.wolfram.com/Notations/>.

Please cite this document by referring to the functions.wolfram.com page from which it was downloaded, for example:

<http://functions.wolfram.com/Constants/E/>

To refer to a particular formula, cite functions.wolfram.com followed by the citation number.

e.g.: <http://functions.wolfram.com/01.03.03.0001.01>

This document is currently in a preliminary form. If you have comments or suggestions, please email comments@functions.wolfram.com.

© 2001-2008, Wolfram Research, Inc.