Note) Slides and example codes are available: https://github.com/mint-lab/3dv_tutorial

An Invitation to 3D Vision: Finding Correspondences

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Table of Contents: Finding Correspondences

- **Feature Points**
  - Q) How can we detect salient points (or parts)?
- **Feature Descriptors**
  - Q) How can we distinguish the points each other?
- **Feature Matching**
  - Q) How can we associate the points across different images?
- **Feature Tracking**
  - Q) How can we associate the points across their next image?
- **Outlier Rejection**
  - Q) How can we select correctly matched points?

Image: pixabay

Getting Started with a Quiz

- **Q) What is it?**

600 pixels

Getting Started with a Quiz

- **Q) What is it?**

600 pixels

Getting Started with a Quiz

- **Q) What is it? "Duck"**

600 pixels   1095 pixels   600 pixels

- **Q) Why corners (junction) instead of edges or blobs?**
  - Human visual systems understand objects better from corners than edges.
  - a.k.a. *feature* points, *key*points, *salient* points, and *interest* points

Getting Started with a Quiz

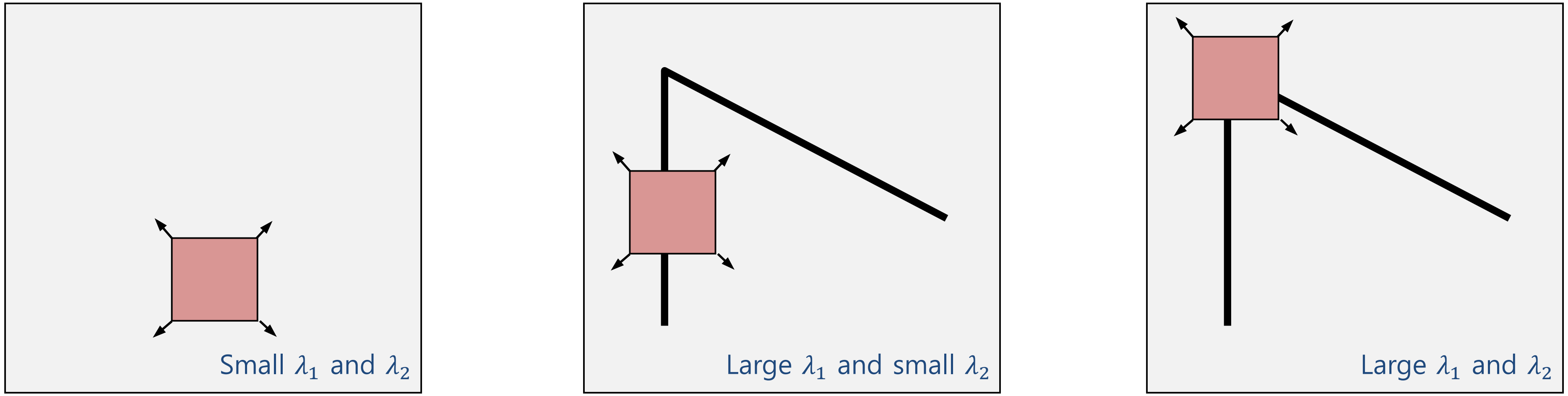
- **Q) Why corners (junction) instead of edges or blobs? Where are the image patches?**

- **Q) What is a good feature? (Requirements)**
  - **Repeatability** (to be invariance/robustness to transformation and noise)
  - **Distinctiveness** (to be easy to distinguish or match)
  - **Locality** (due to occlusion)
  - Quantity (sufficient number), accuracy (localization), efficiency (computing time), …

Image: [OpenCV Tutorials]

Feature Point) Harris Corner (1988)

- Key idea: **Sliding window**



"flat" region:
no change
in all directions

"edge":
no change
along the edge direction

"corner":
significant change
in all directions

– Formulation

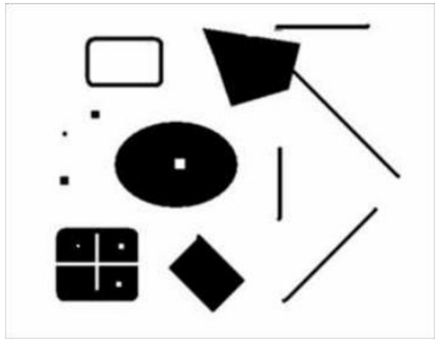
- $I\left(x+\Delta_x, y+\Delta_y\right) \approx I(x, y) + [I_x(x,y) \quad I_y(x,y)] \begin{bmatrix} \Delta_x \\ \Delta_y \end{bmatrix}$ where $I_x = \frac{\partial I}{\partial x}$ and $I_y = \frac{\partial I}{\partial y}$
- $D\left(\Delta_x, \Delta_y\right) = \sum_{(x,y)\in W} \left(I\left(x+\Delta_x, y+\Delta_y\right) - I(x,y)\right)^2 \approx [\Delta_x \quad \Delta_y] \underbrace{\begin{bmatrix} \sum_W I_x^2 & \sum_W I_x I_y \\ \sum_W I_x I_y & \sum_W I_y^2 \end{bmatrix}}_{\mathrm{M}} \begin{bmatrix} \Delta_x \\ \Delta_y \end{bmatrix}$
- Two eigen values of M: λ_1 and λ_2

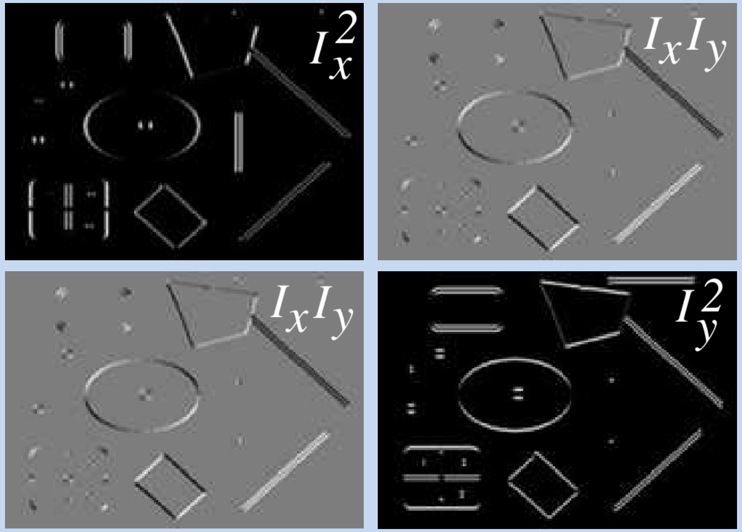
Image: Frolova and Simakov, "Matching with Invariant Features" (Lecture Slides), 2004

Feature Point) Harris Corner (1988)

- Key idea: **Sliding window**
  - Formulation
    - $I(x+\Delta_x, y+\Delta_y) \approx I(x,y) + [I_x(x,y) \quad I_y(x,y)] \begin{bmatrix} \Delta_x \\ \Delta_y \end{bmatrix}$ where $I_x = \frac{\partial I}{\partial x}$ and $I_y = \frac{\partial I}{\partial y}$
    - $D(\Delta_x, \Delta_y) = \sum_{(x,y)\in W} \left(I(x+\Delta_x, y+\Delta_y) - I(x,y) \right)^2 \approx [\Delta_x \quad \Delta_y] \underbrace{\begin{bmatrix} \sum_W I_x^2 & \sum_W I_x I_y \\ \sum_W I_x I_y & \sum_W I_y^2 \end{bmatrix}}_{\mathrm{M}} \begin{bmatrix} \Delta_x \\ \Delta_y \end{bmatrix}$
  - **Harris corner response**
    - $\text{cornerness} = \det(\mathrm{M}) - k \operatorname{trace}(\mathrm{M})^2$
    - Note) $\det(\mathrm{M}) = \lambda_1 \lambda_2$, $\operatorname{trace}(\mathrm{M}) = \lambda_1 + \lambda_2$, and $k \in [0.04, 0.06]$
  - Note) **Good-Feature-to-Track** (a.k.a. GFTT or Shi-Tomasi corner; 1994)
    - $\text{cornerness} = \min(\lambda_1, \lambda_2)$

$I(x,y)$





M

$\det(\mathrm{M}) - k \operatorname{trace}(\mathrm{M})^2$

Image: Tuytelaars et al., "Local Invariant Feature Detectors: A Survey", Foundations and Trends in Computer Graphics and Vision, 2008 DOI

Feature Point) Harris Corner (1988)

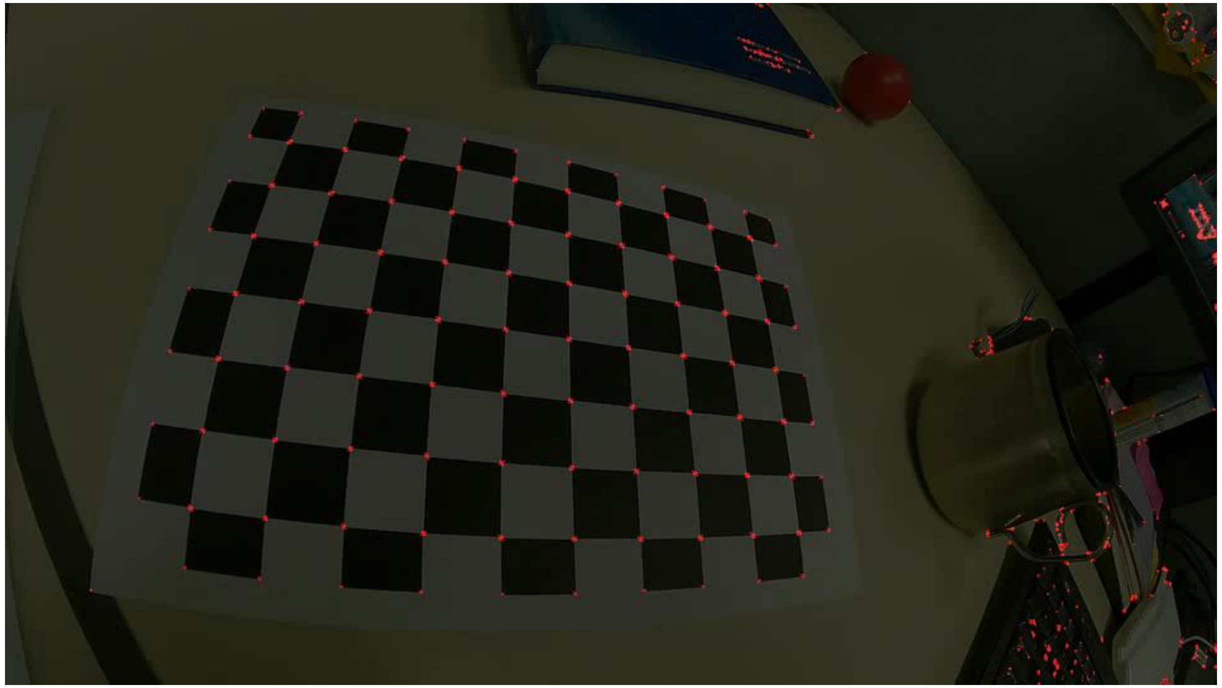
- Example) **Harris corner implementation** [harris_corner_implement.py]

```
def cornerHarris(img, ksize=3, k=0.04):
    # Compute gradients and M matrix
    Ix = cv.Sobel(img, cv.CV_32F, 1, 0)
    Iy = cv.Sobel(img, cv.CV_32F, 0, 1)
    M11 = cv.GaussianBlur(Ix*Ix, (ksize, ksize), 0)
    M22 = cv.GaussianBlur(Iy*Iy, (ksize, ksize), 0)
    M12 = cv.GaussianBlur(Ix*Iy, (ksize, ksize), 0)

    # Compute Harris cornerness
    detM = M11 * M22 - M12 * M12
    traceM = M11 + M22
    cornerness = detM - k * traceM**2
    return cornerness
```

$$\mathrm{M} = \begin{bmatrix} \sum_W I_x^2 & \sum_W I_x I_y \\ \sum_W I_x I_y & \sum_W I_y^2 \end{bmatrix}$$

$$\text{cornerness} = \det(\mathrm{M}) - k \,\mathrm{trace}(\mathrm{M})^2$$



Feature Point) Harris Corner (1988)

- Properties
  - Invariant to translation, rotation, and intensity shift ($I \rightarrow I + b$) ~~intensity scaling ($I \rightarrow aI$)~~
  - Variant to **image scaling**

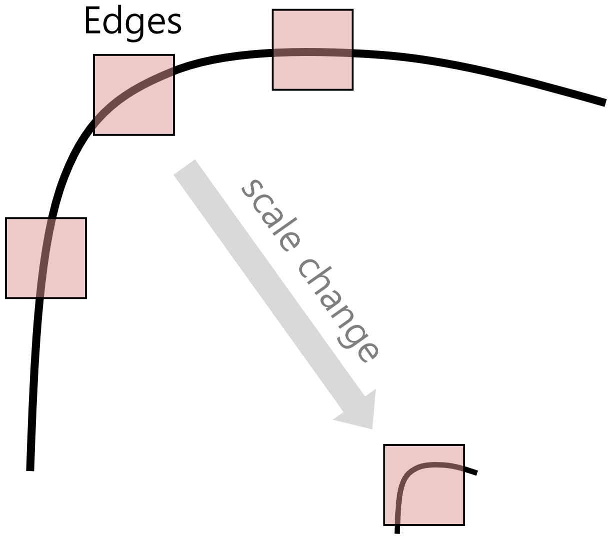
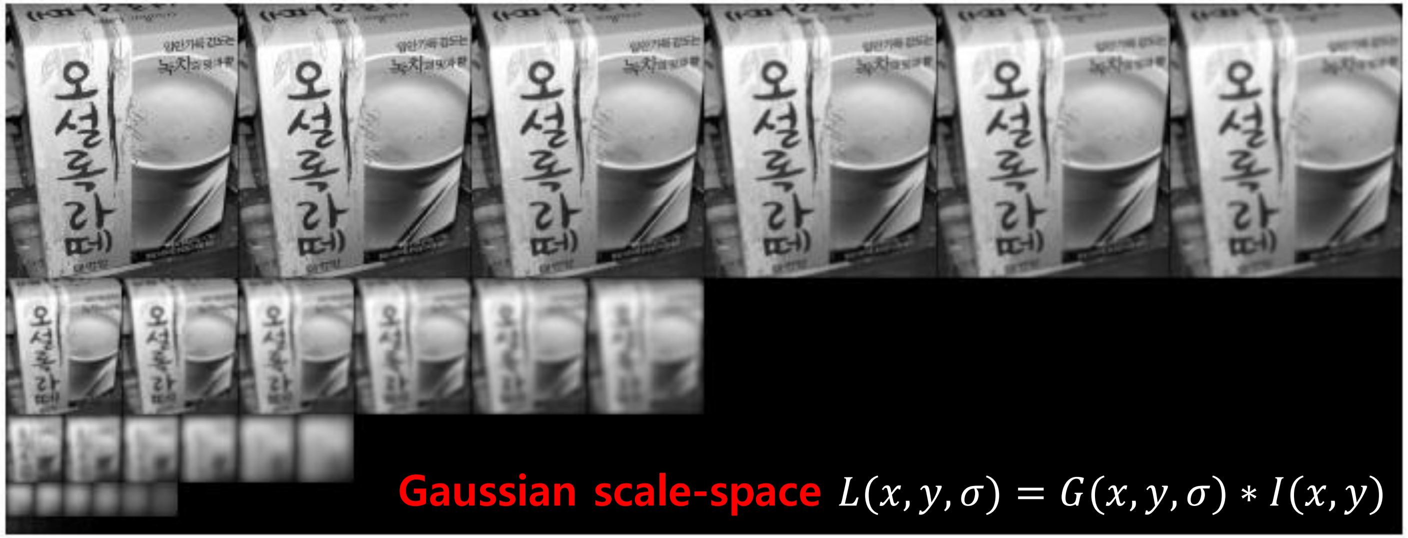
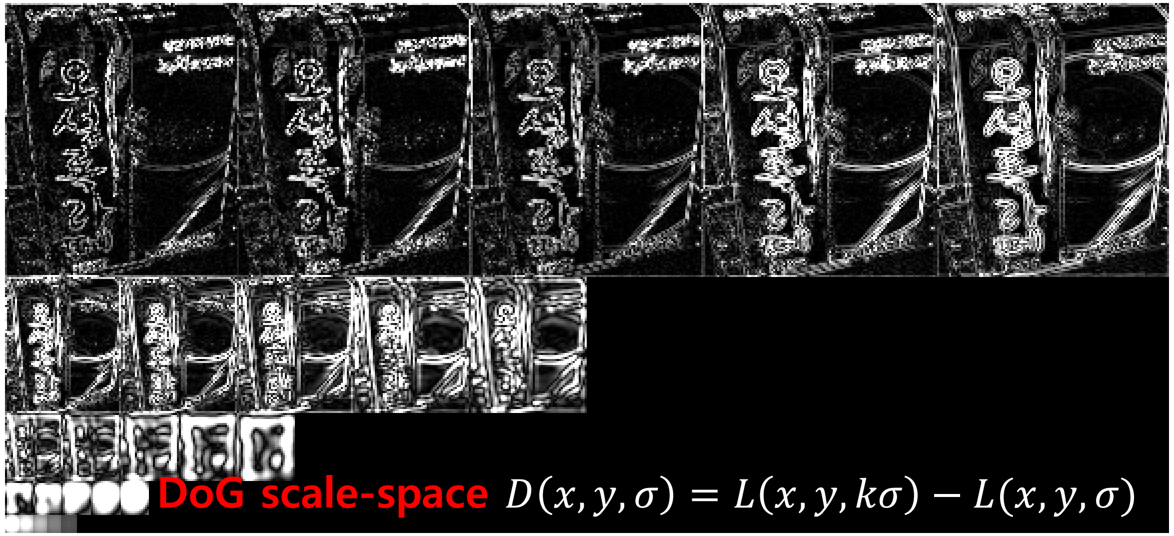


Image: Frolova and Simakov, "Matching with Invariant Features" (Lecture Slides), 2004

Feature Point) SIFT (Scale-Invariant Feature Transform; 1999)

- Key idea: **Scale-space** (~ image pyramid) **and DoG** (difference of Gaussian)



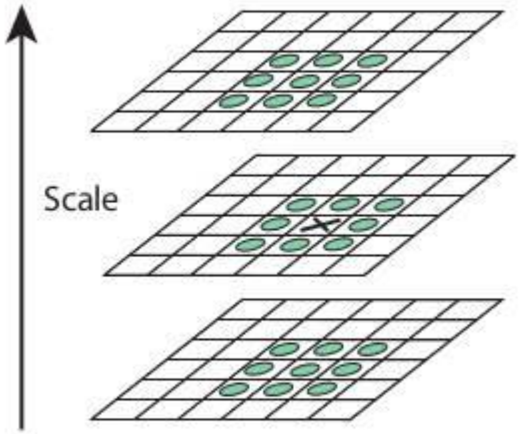


Note) The patent for SIFT feature and descriptors was expired at March 6, 2020.

Feature Point) SIFT (Scale-Invariant Feature Transform; 1999)

- Key idea: **Scale-space** (~ image pyramid) **and DoG** (difference of Gaussian)
- Part #1) **Feature point detection**
  1. Find **local extrema** (minima and maxima) in DoG scale-space
  2. Localize their position accurately (sub-pixel level) using 3D quadratic function
  3. Eliminate **low contrast candidates**, $|D(\mathbf{x})| < \tau$
  4. Eliminate **candidates on edges**,

$$\frac{\mathrm{trace}(H)^2}{\det(H)} < \frac{(r+1)^2}{r} \quad \text{where} \quad H = \begin{bmatrix} D_{xx} & D_{xy} \\ D_{xy} & D_{yy} \end{bmatrix}$$



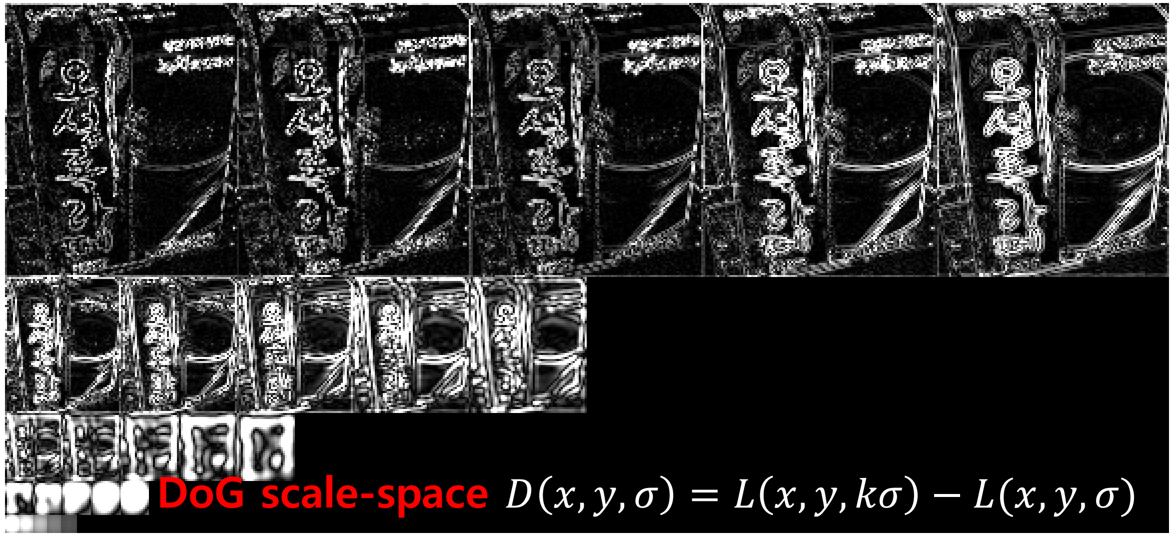


Image: David Lowe, "Distinctive Image Features from Scale-invariant Keypoints", IJCV, 2004 DOI PDF

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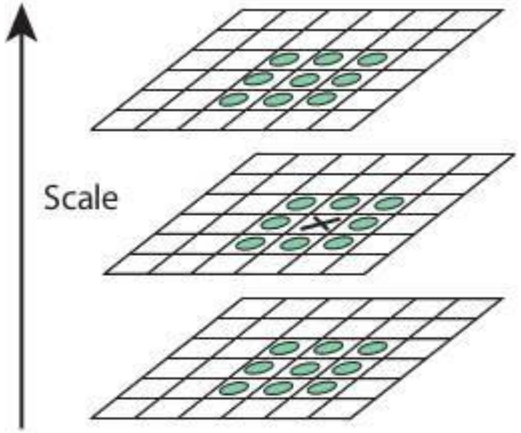
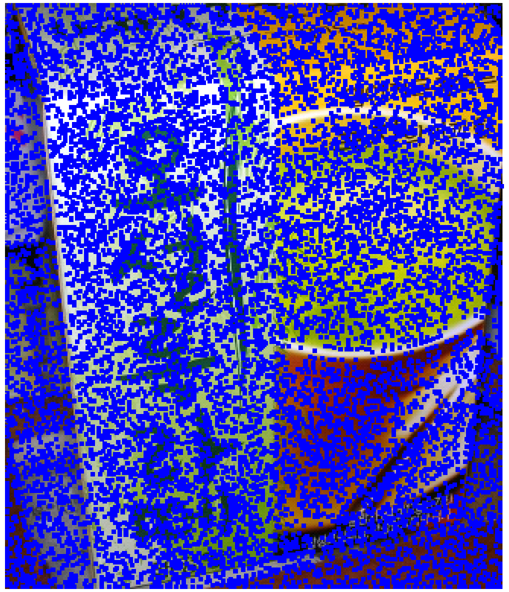


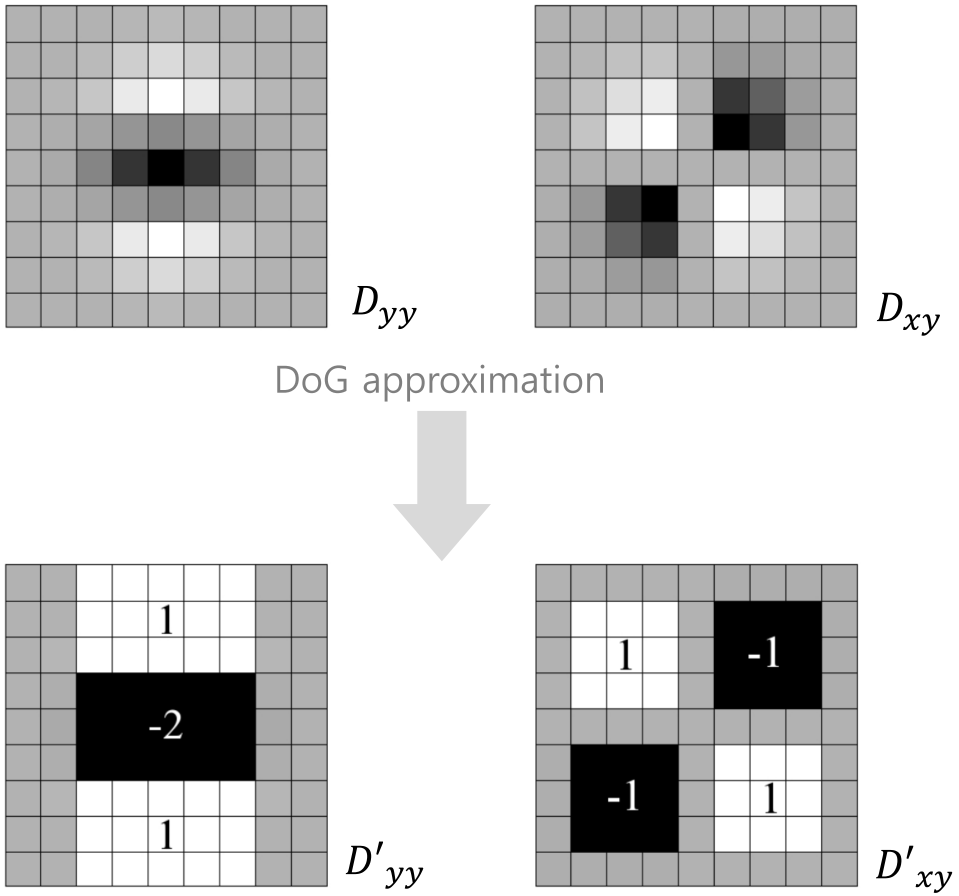


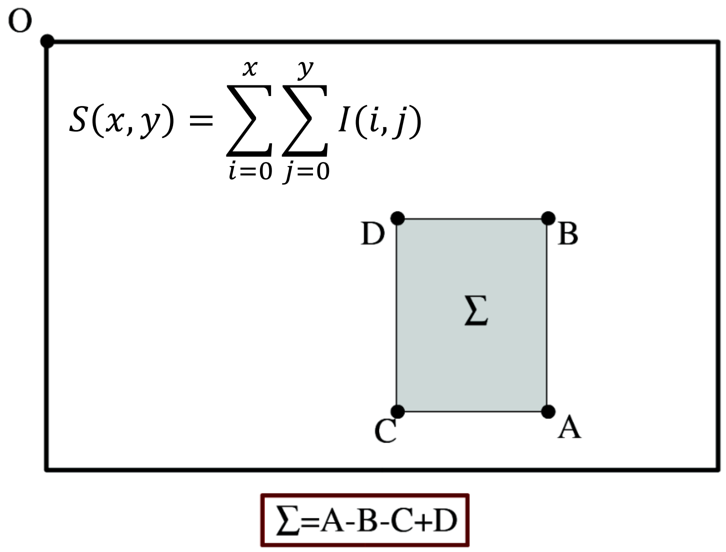
Local extrema (N: 11479)

Feature points (N: 971)

Feature Point) SURF (Speeded Up Robust Features; 2006)

- Key idea: **Approximation of SIFT**
  - e.g. DoG approximation Haar-like features and **integral image**





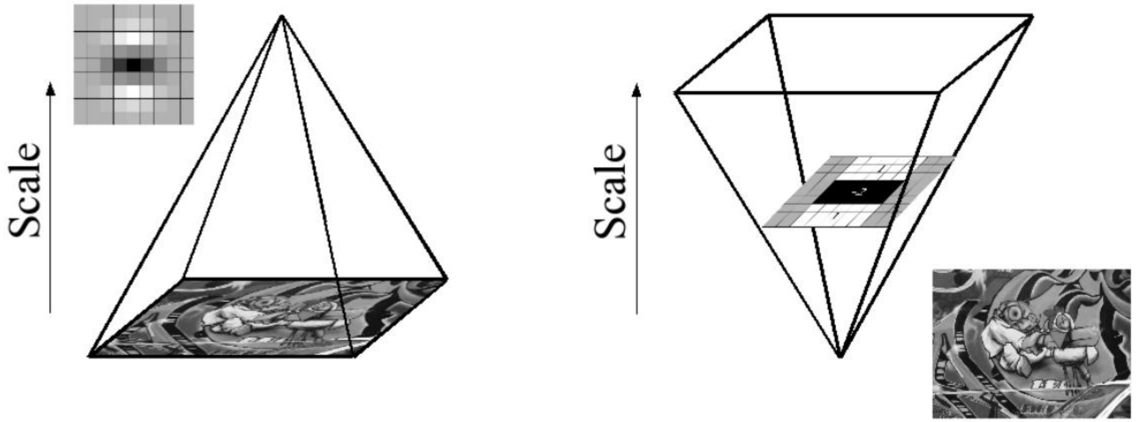


Image: Bay et. al., "Speeded Up Robust Features (SURF)", CVIU, 2008 DOI

Feature Descriptor) **SIFT (Scale-Invariant Feature Transform; 1999)**

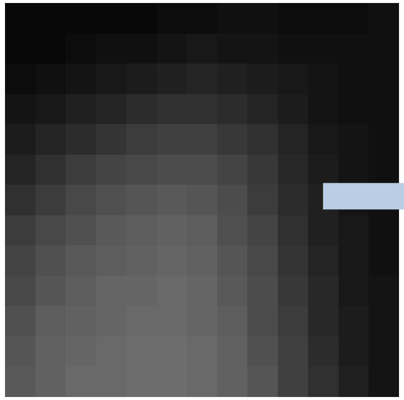
- Part #2) **Orientation assignment**
  1. Derive magnitude and orientation of gradient of each patch

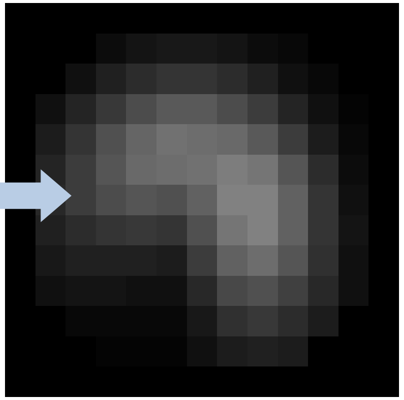
$$m(x,y) = \sqrt{\left(L(x+1,y) - L(x-1,y)\right)^2 + \left(L(x,y+1) - L(x,y-1)\right)^2}$$

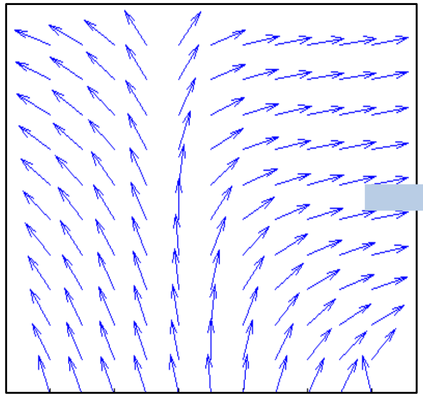
$$\theta(x,y) = \tan^{-1}\frac{L(x,y+1) - L(x,y-1)}{L(x+1,y) - L(x-1,y)}$$

  2. Find the strongest orientation
     - Histogram voting (36 bins) with Gaussian-weighted magnitude

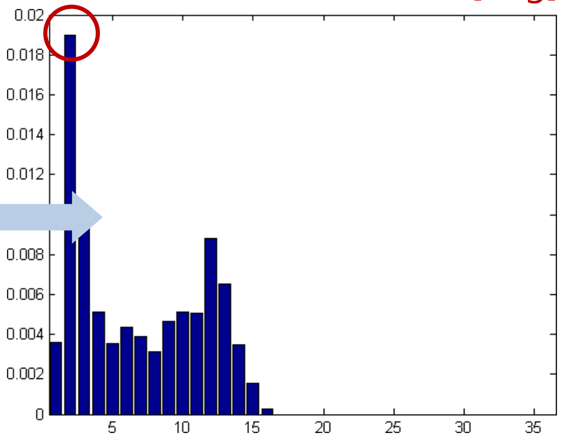

Feature scales and orientations


feature patch


gradient magnitude
$m(x,y)$


gradient orientation
$\theta(x,y)$

feature orientation ≈ 15 [deg]

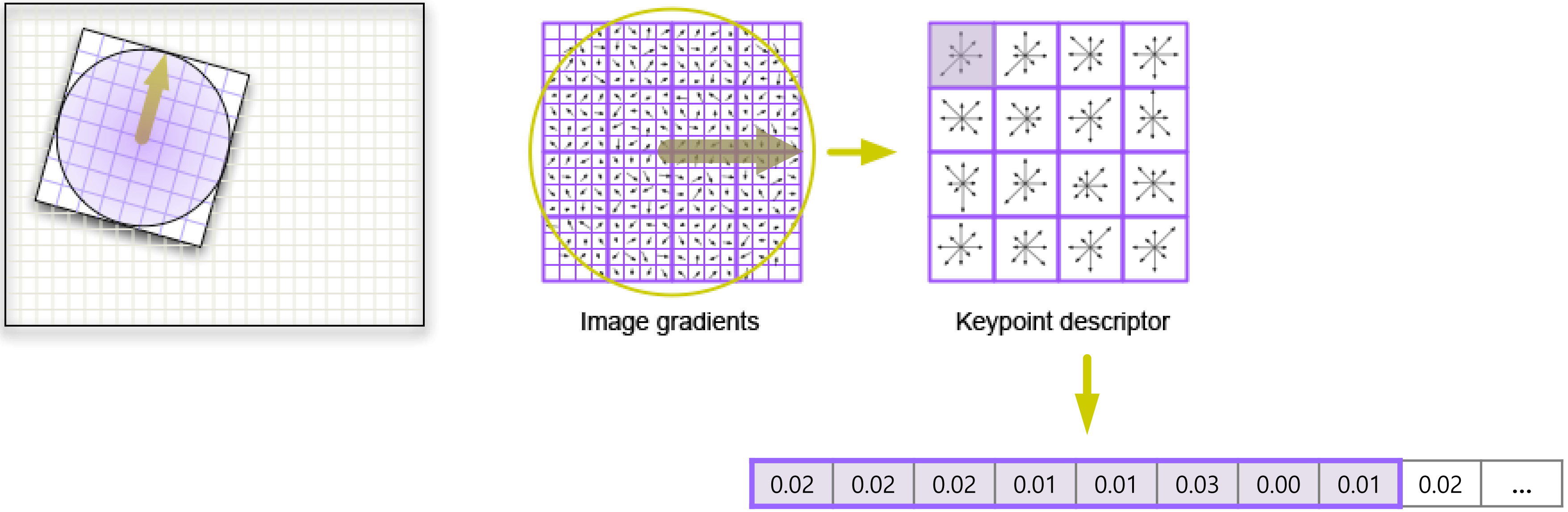


histogram voting (36 bins)

Feature Descriptor) **SIFT (Scale-Invariant Feature Transform; 1999)**

Feature scales and orientations

- Part #3) **Feature descriptor extraction**
  1. Build a 4x4 gradient histogram (8 bins) from each patch (16x16 pixels)
     - Use Gaussian-weighted magnitude again
     - Use relative angles w.r.t. the assigned feature orientation
  2. Encode the histogram into a 128-dimensional vector
     - Apply normalization to be an unit vector

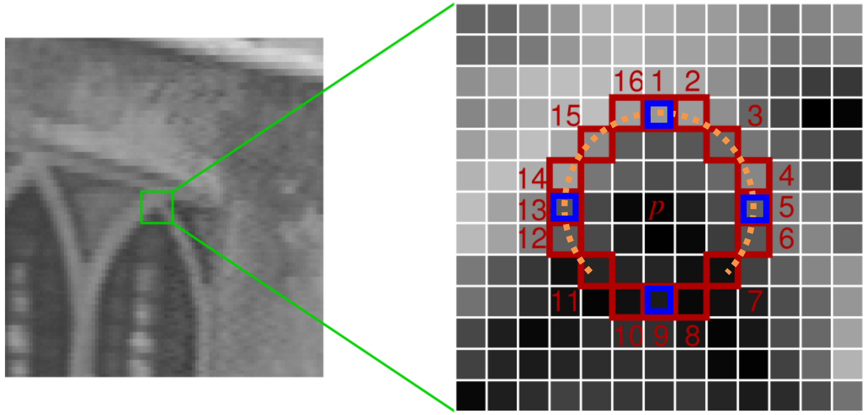


Keypoint descriptor (dim: 128)

Image: Jonas Hurrelmann, “Harris Corner Detector & SIFT” (Lecture Slides), 2006

Feature Point) FAST (Features from Accelerated Segment Test; 2006)

- Key idea: **Intensity check of continuous arc of n pixels**
  - Is this point p a corner? (I_p: intensity at p, t: the intensity threshold)
    - Is a segment of n continuous pixels brighter than $I_p + t$? (OR) Is the segment darker than $I_p - t$?
    - Note) High-speed non-corner rejection: Checking intensity values at 1, 9, 5, and 13 (n: 12)



  - Too many corners! → Non-maximum suppression
- Versions
  - FAST-9 (n: 9; `cv.FastFeatureDetector_TYPE_9_16`), FAST-12 (n: 12), …
  - FAST-**ER**: Training a decision tree to **e**nhance **r**epeatability with more pixels

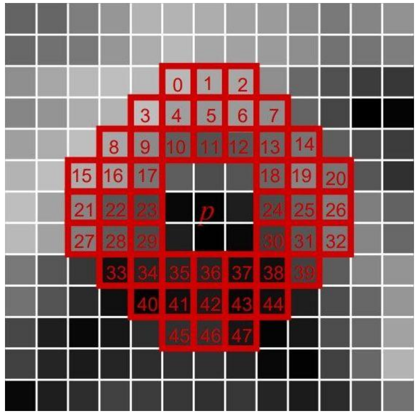
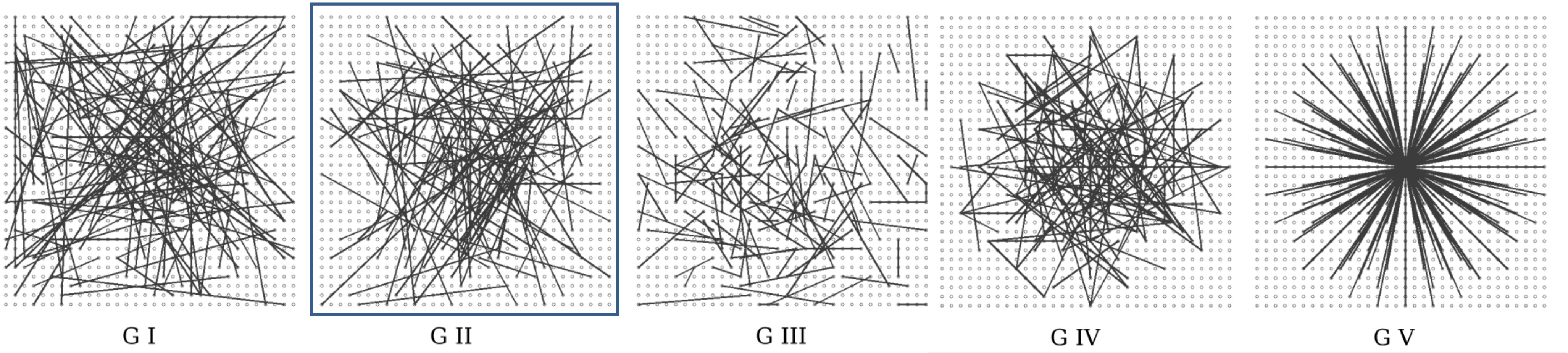


Image: Rosten et al., "Faster and Better: A Machine Learning Approach to Corner Detection", T-PAMI, 2010 DOI

Feature Descriptor) BRIEF (Binary Robust Independent Elementary Features; 2010)

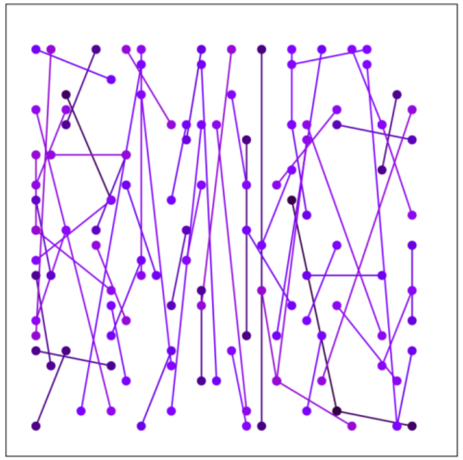
- Key idea: **Intensity comparison of a sequence of random pairs (binary test)**
  - Path size: 31 x 31 pixels (Note: Applying smoothing for stability and repeatability)



  - Descriptor size: 128 tests (128 bits) → 16 bytes
    - Note) SIFT: 128-dimensional vector → 512 bytes
- Versions: The number of tests
  - BRIEF-32, BRIEF-64, BRIEF-128 (16 bytes), BRIEF-256 (32 bytes), BRIEF-512 (64 bytes), …
- Combination examples
  - SIFT feature points + BRIEF descriptors
  - FAST feature points + BRIEF descriptors

Image: Calonder et. al., "BRIEF: Computing a Local Binary Descriptor Very Fast", T-PAMI, 2012 DOI PDF

ORB (Oriented FAST and rotated BRIEF, 2011)



- Key idea: **Adding rotation invariance to BRIEF**
  - **Oriented FAST**
    - Generate scale pyramid for scale invariance
    - Detect *FAST-9* points (filtering with Harris corner response)
    - Calculate feature orientation by *intensity centroid* $C = \left(\frac{m_{10}}{m_{00}}, \frac{m_{01}}{m_{00}}\right)$

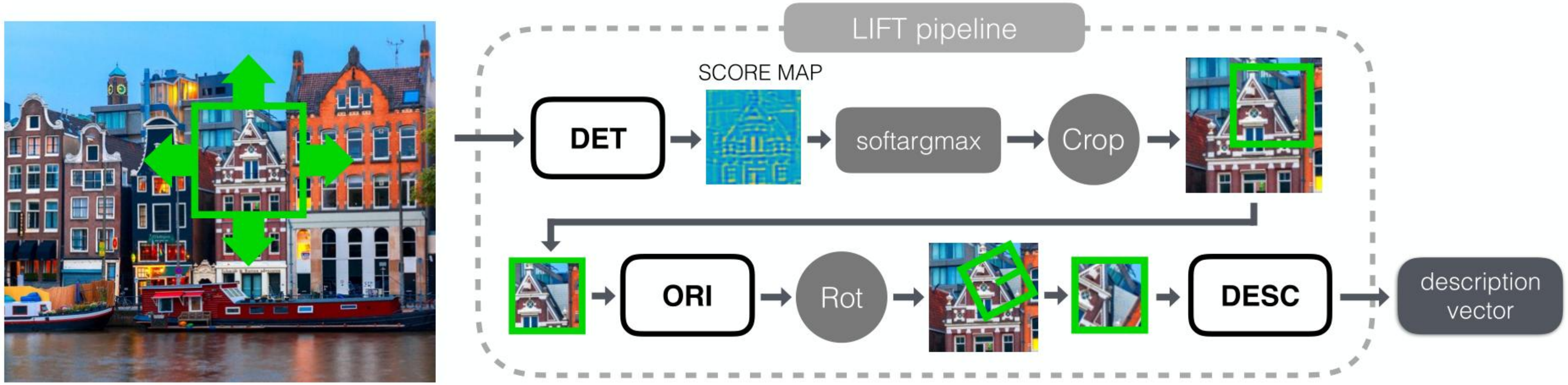
$$\theta = \tan^{-1}\frac{m_{01}}{m_{10}} \quad \text{where} \quad m_{pq} = \sum_{x,y} x^p y^q I(x,y)$$

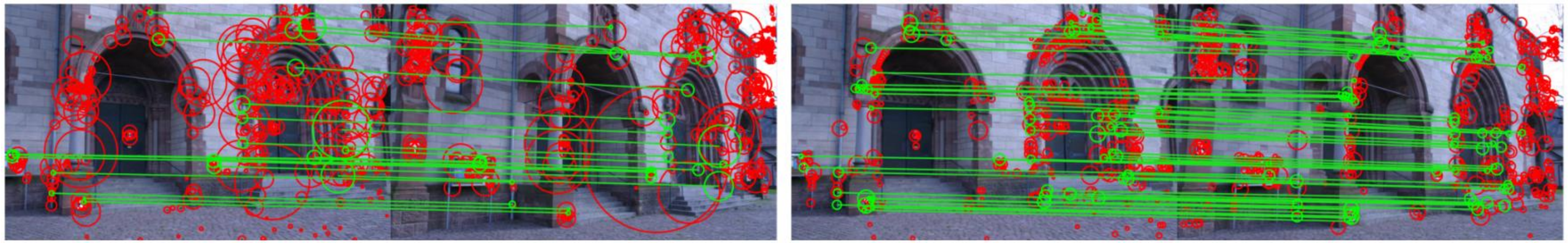
  - **Rotation-aware BRIEF**
    - Extract BRIEF descriptors w.r.t. the known orientation
    - Use better comparison pairs trained by greedy search
- Combination (default): **ORB**
  - FAST-9 detector (with orientation) + BRIEF-256 descriptor (with trained pairs)
- Computing time
  - ORB: **15.3 [msec]** / SURF: 217.3 [msec] / SIFT: 5228.7 [msec] @ 24 images (640x480) in Pascal dataset

Image: Rublee et. al., "ORB: An Efficient Alternative to SIFT or SURF", ICCV, 2011 DOI

Feature Point and Descriptor) LIFT (Learned Invariant Feature Transform; 2016)

- Key idea: **Deep neural network**
  - CNN network: **DET** (feature detector) + **ORI** (orientation estimator) + **DESC** (feature descriptor)
  - Training data: Photo Tourism dataset with VisualSFM (SIFT)





SIFT LIFT

Image: Yi et. al., "LIFT: Learned Invariant Feature Transform", ECCV, 2016 arXiv Github

Feature Point and Descriptor) SuperPoint (2017)

- Key idea: **Self-supervised training with homography transformation**
  - CNN network: Encoder (~ VGG) + Decoders (for point and descriptor)

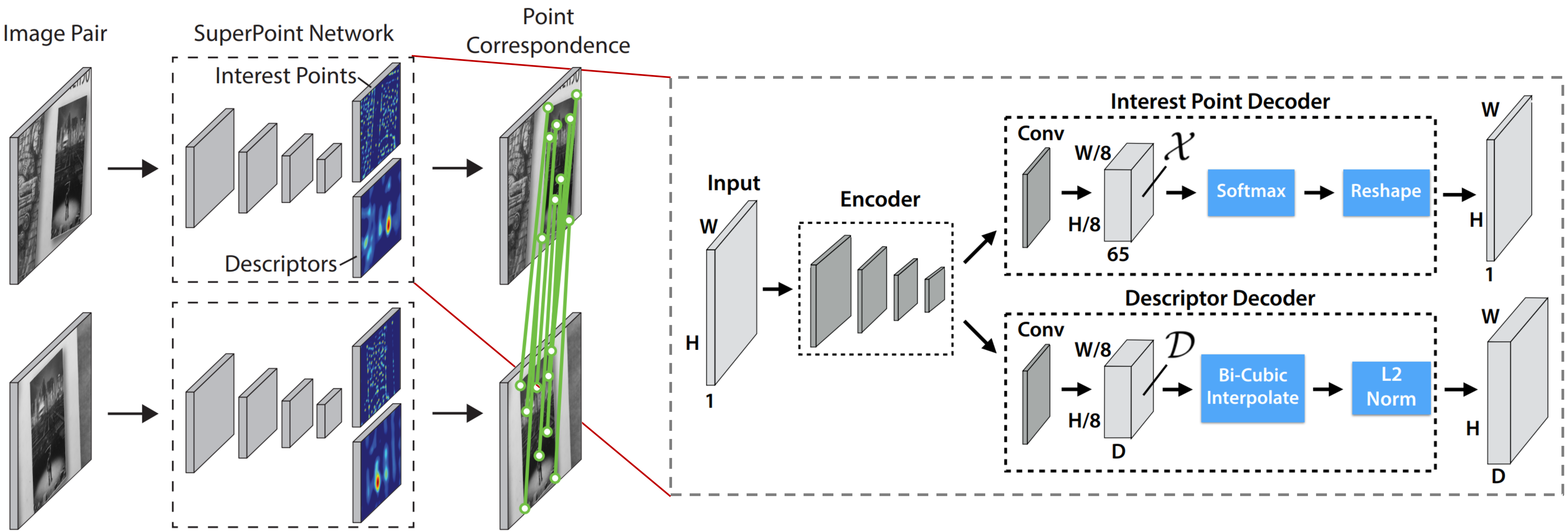
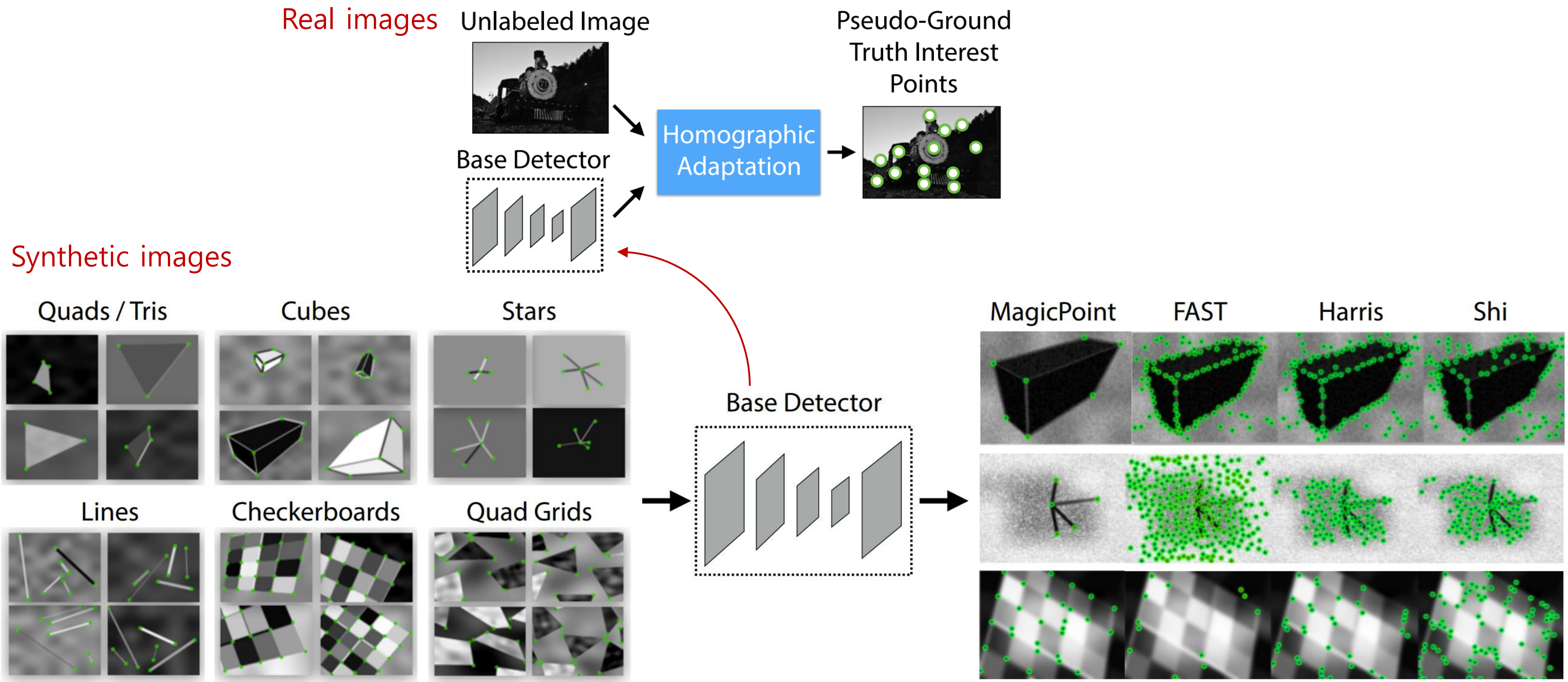


Image: DeTone et. al., "SuperPoint: Self-Supervised Interest Point Detection and Description", CVPRW, 2018 arXiv Github (authors) Github

Feature Point and Descriptor) SuperPoint (2017)

- Key idea: **Self-supervised training with homography transformation**
  - Training data generation: The base detector, *MagicPoint* → Ground truth interest points



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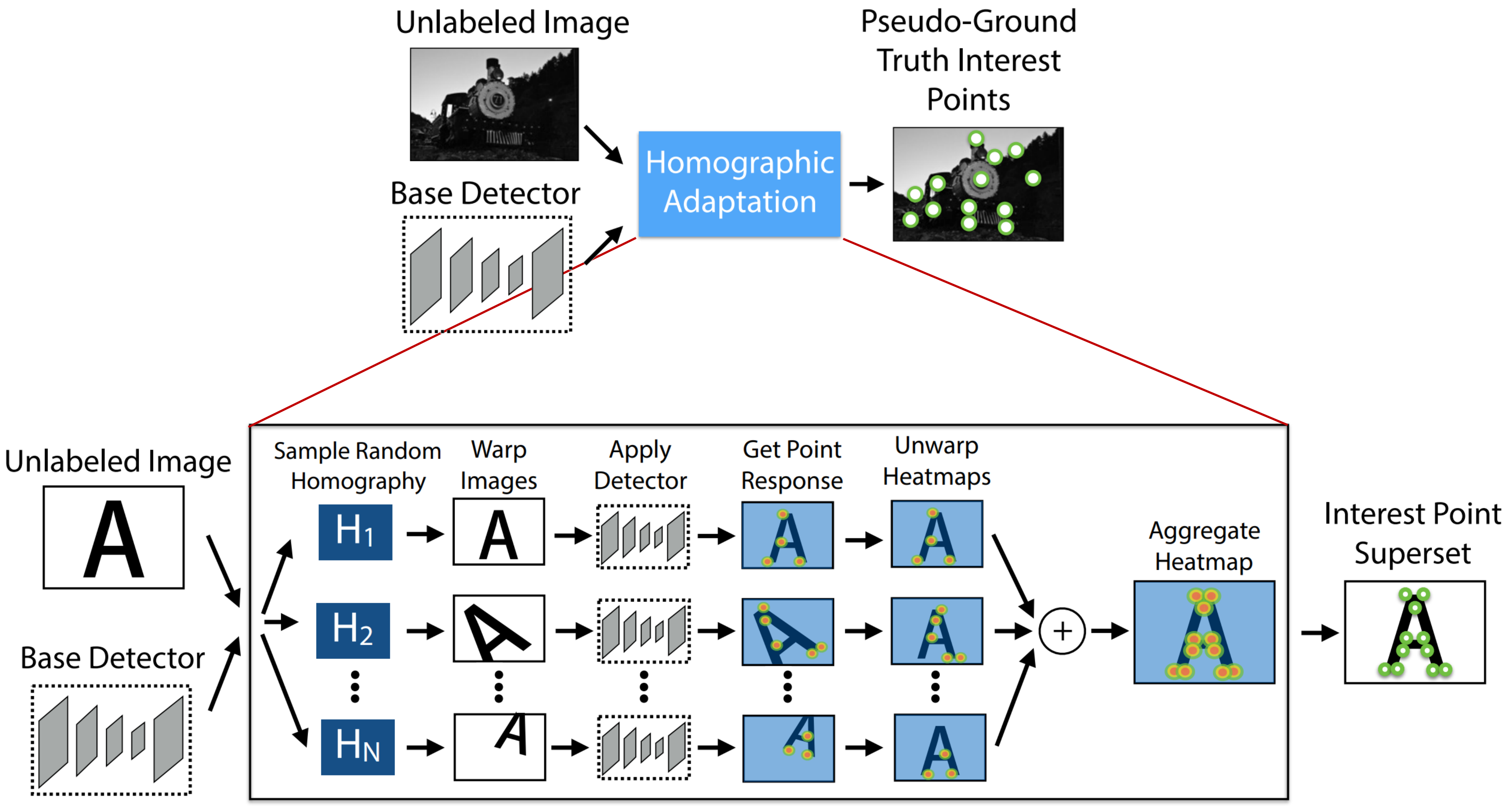


Image: DeTone et. al., "SuperPoint: Self-Supervised Interest Point Detection and Description", CVPRW, 2018 arXiv Github (authors) Github

Feature Point and Descriptor) SuperPoint (2017)

- Key idea: **Self-supervised training with homography transformation**
  - Training data augmentation: Random homography transformation
  - Loss functions: Interest Point Loss (X, Y) + Interest Point Loss (X', Y') + Descriptor Loss (Y, Y')

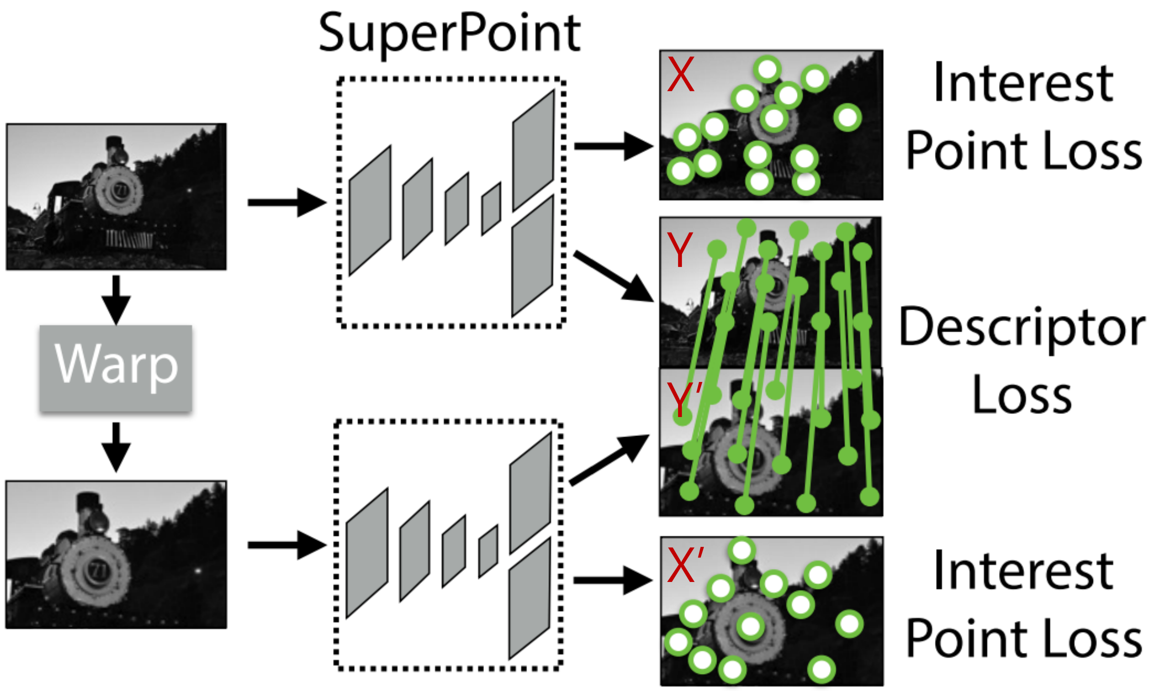
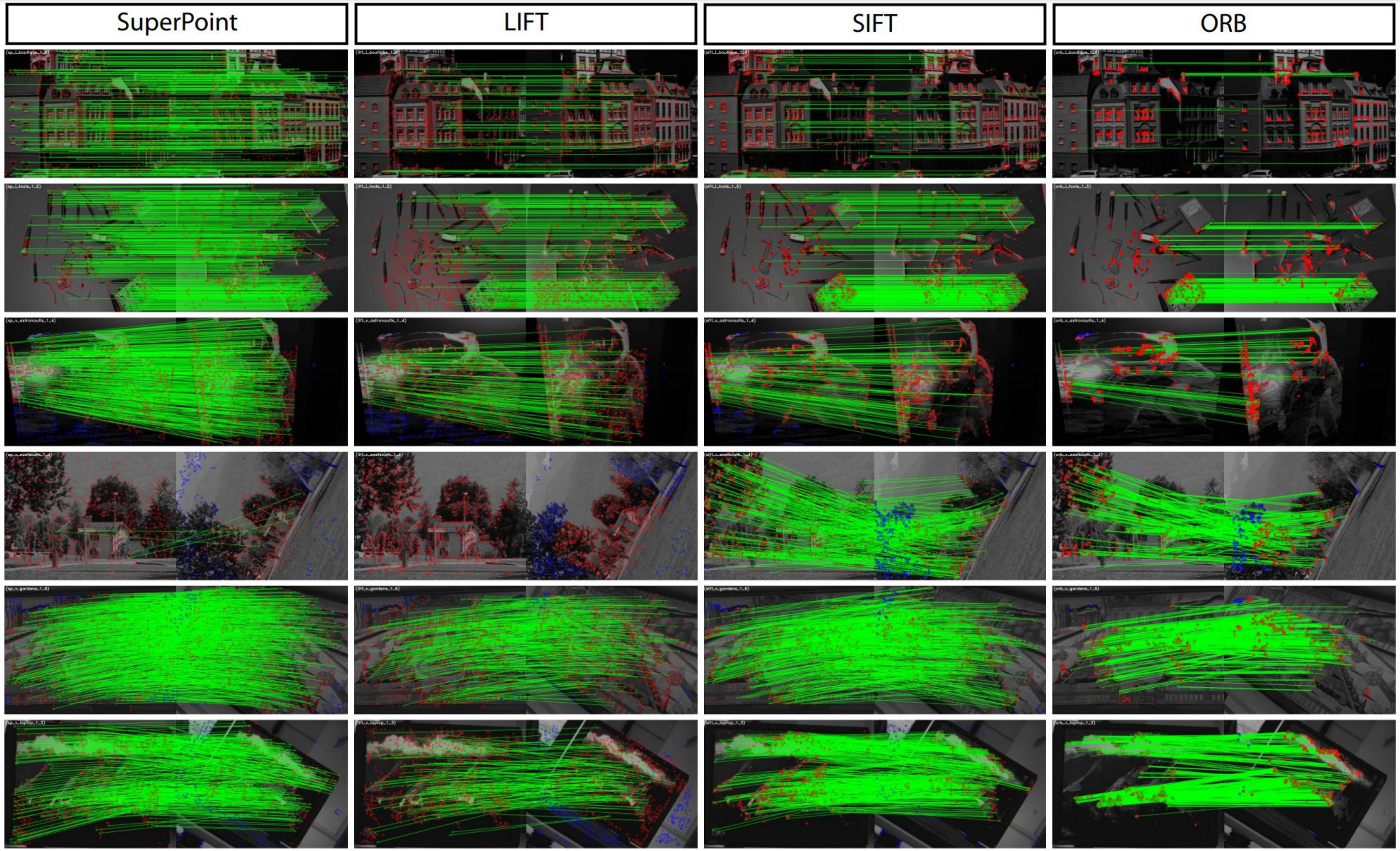


Image: DeTone et. al., "SuperPoint: Self-Supervised Interest Point Detection and Description", CVPRW, 2018 arXiv Github (authors) Github

Feature Point and Descriptor) SuperPoint (2017)

- Key idea: **Self-supervised training with homography transformation**
  - Real-time performance: **70 FPS** (13 msec) on 640x480 images with NVIDIA Titan X GPU



Freiburg RGBD:

Microsoft 7 Scenes:

MonoVO:

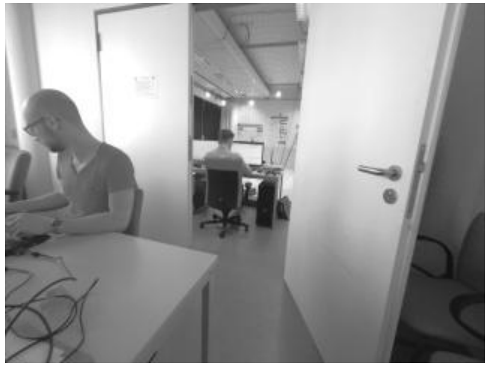
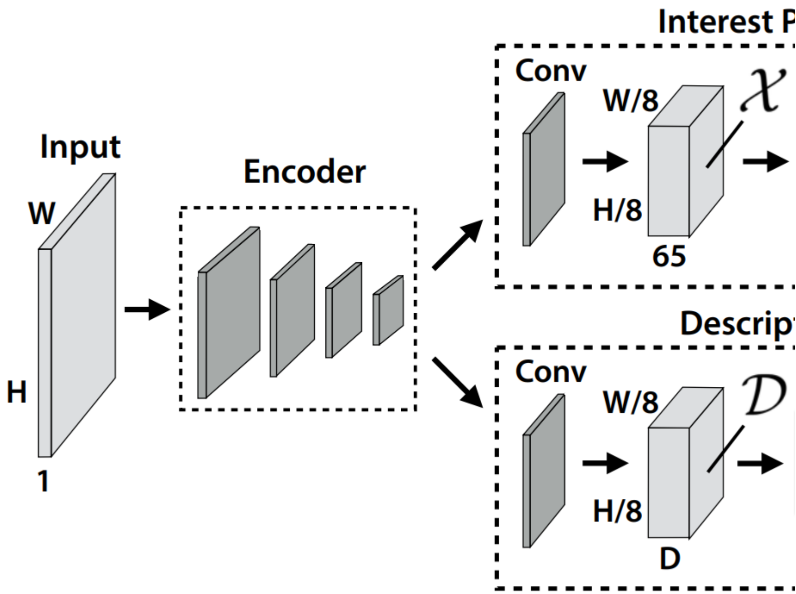


Image: DeTone et. al., “SuperPoint: Self-Supervised Interest Point Detection and Description”, CVPRW, 2018 arXiv Github (authors) Github

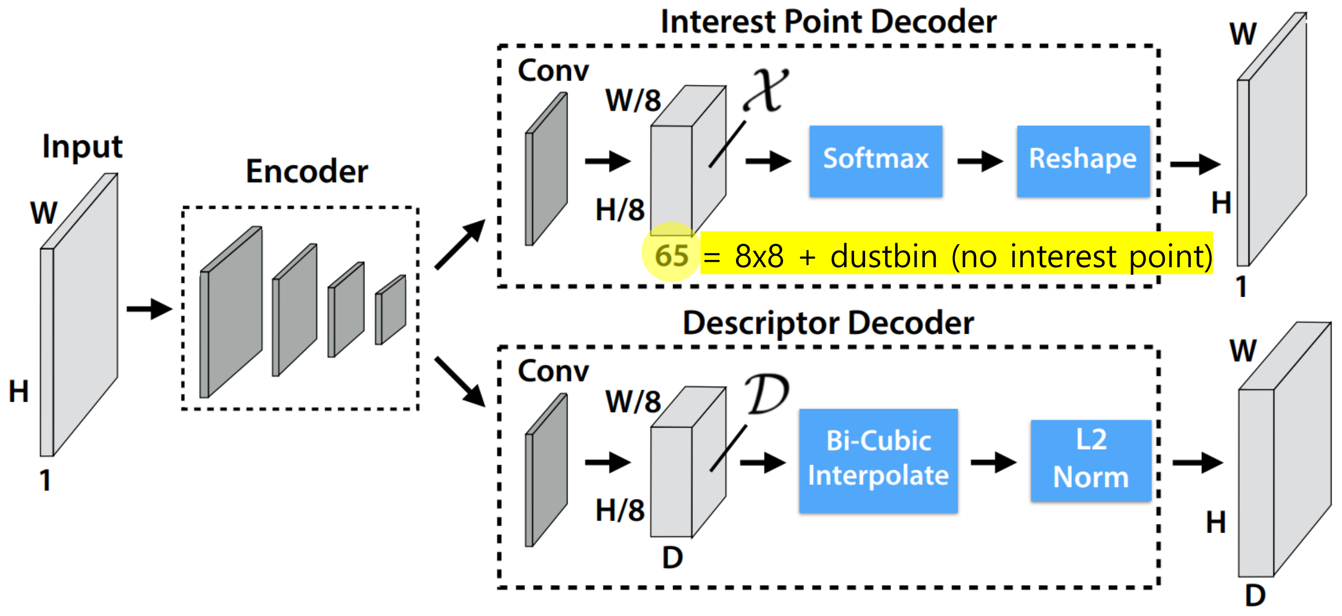
Feature Point and Descriptor) **SuperPoint (2017)**

- Example) **SuperPoint implementation** [demo_superpoint.py] @ Github (authors)

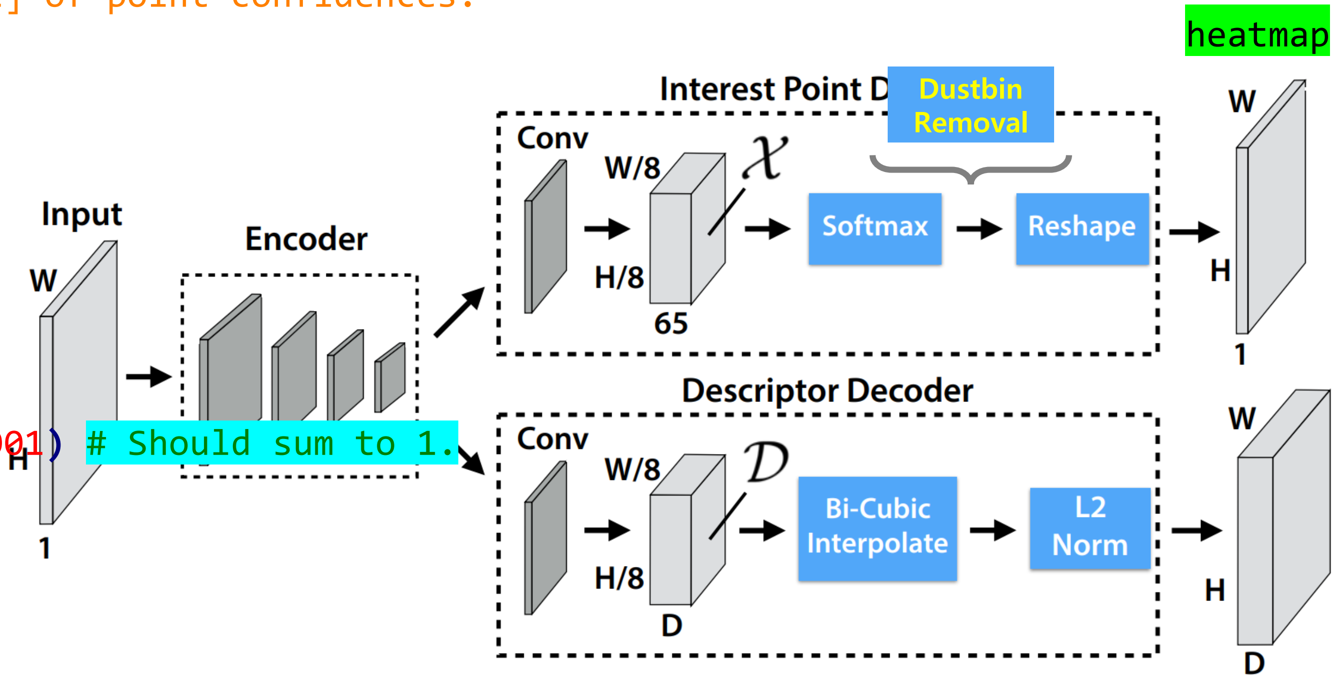
```
class SuperPointNet(torch.nn.Module):
  """ Pytorch definition of SuperPoint Network. """
  def __init__(self):
    super(SuperPointNet, self).__init__()
    self.relu = torch.nn.ReLU(inplace=True)
    self.pool = torch.nn.MaxPool2d(kernel_size=2, stride=2)
    c1, c2, c3, c4, c5, d1 = 64, 64, 128, 128, 256, 256
    # Shared Encoder.
    self.conv1a = torch.nn.Conv2d(1, c1, kernel_size=3, stride=1, padding=1)
    self.conv1b = torch.nn.Conv2d(c1, c1, kernel_size=3, stride=1, padding=1)
    self.conv2a = torch.nn.Conv2d(c1, c2, kernel_size=3, stride=1, padding=1)
    self.conv2b = torch.nn.Conv2d(c2, c2, kernel_size=3, stride=1, padding=1)
    self.conv3a = torch.nn.Conv2d(c2, c3, kernel_size=3, stride=1, padding=1)
    self.conv3b = torch.nn.Conv2d(c3, c3, kernel_size=3, stride=1, padding=1)
    self.conv4a = torch.nn.Conv2d(c3, c4, kernel_size=3, stride=1, padding=1)
    self.conv4b = torch.nn.Conv2d(c4, c4, kernel_size=3, stride=1, padding=1)
    # Detector Head.
    self.convPa = torch.nn.Conv2d(c4, c5, kernel_size=3, stride=1, padding=1)
    self.convPb = torch.nn.Conv2d(c5, 65, kernel_size=1, stride=1, padding=0)
    # Descriptor Head.
    self.convDa = torch.nn.Conv2d(c4, c5, kernel_size=3, stride=1, padding=1)
    self.convDb = torch.nn.Conv2d(c5, d1, kernel_size=1, stride=1, padding=0)
```



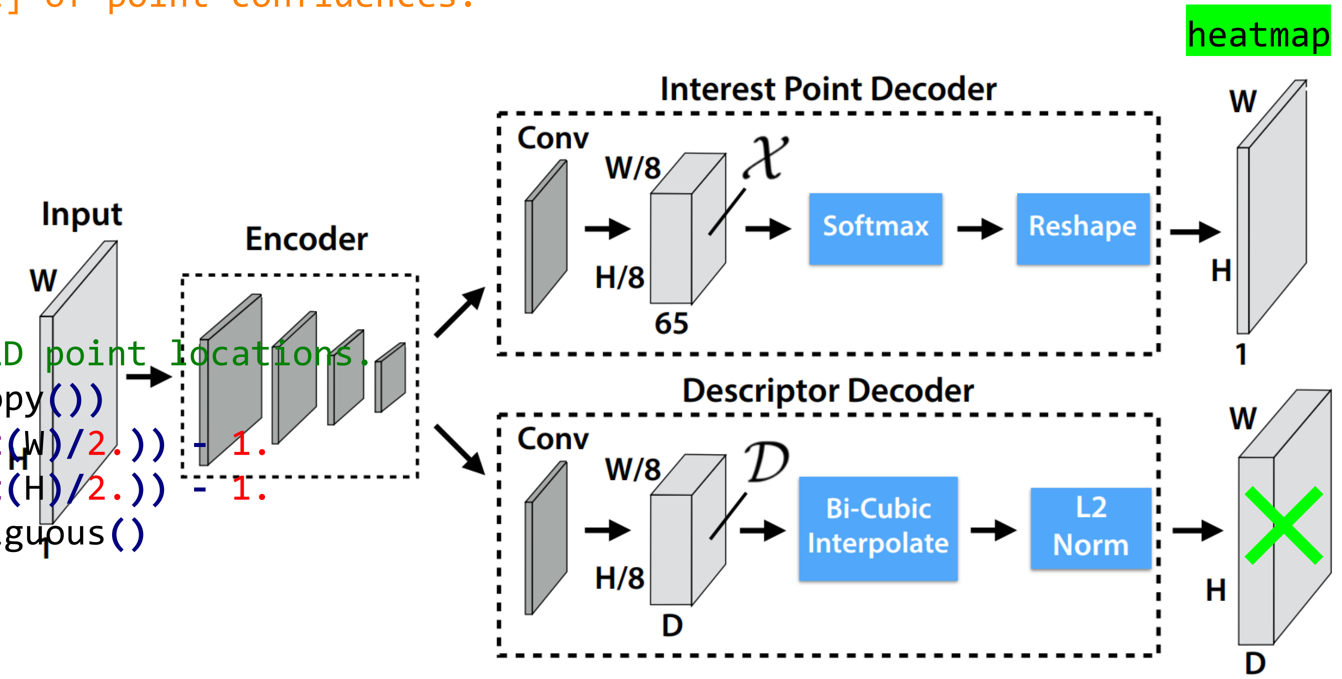
```python
class SuperPointNet(torch.nn.Module):
  ...
  def forward(self, x):
    """ Forward pass that jointly computes unprocessed point and descriptor
    tensors.
    Input
      x: Image pytorch tensor shaped N x 1 x H x W.
    Output
      semi: Output point pytorch tensor shaped N x 65 x H/8 x W/8. (Note: 65 channels includes a dustbin.)
      desc: Output descriptor pytorch tensor shaped N x 256 x H/8 x W/8.
    """
    # Shared Encoder.
    x = self.relu(self.conv1a(x))
    x = self.relu(self.conv1b(x))
    x = self.pool(x)
    x = self.relu(self.conv2a(x))
    x = self.relu(self.conv2b(x))
    x = self.pool(x)
    x = self.relu(self.conv3a(x))
    x = self.relu(self.conv3b(x))
    x = self.pool(x)
    x = self.relu(self.conv4a(x))
    x = self.relu(self.conv4b(x))
    # Detector Head.
    cPa = self.relu(self.convPa(x))
    semi = self.convPb(cPa)
    # Descriptor Head.
    cDa = self.relu(self.convDa(x))
    desc = self.convDb(cDa)
    dn = torch.norm(desc, p=2, dim=1) # Compute the norm.
    desc = desc.div(torch.unsqueeze(dn, 1)) # Divide by norm to normalize.
    return semi, desc
```



```python
class SuperPointNet(torch.nn.Module):
  ...
  def run(self, img):
    """ Process a numpy image to extract points and descriptors.
    Input
      img - HxW numpy float32 input image in range [0,1].
    Output
      corners - 3xN numpy array with corners [x_i, y_i, confidence_i]^T.
      desc - 256xN numpy array of corresponding unit normalized descriptors.
      heatmap - HxW numpy heatmap in range [0,1] of point confidences.
    """
    ...
    # Forward pass of network.
    outs = self.net.forward(inp)
    semi, coarse_desc = outs[0], outs[1]
    # Convert pytorch -> numpy.
    semi = semi.data.cpu().numpy().squeeze()
    # --- Process points.
    dense = np.exp(semi) # Softmax.
    dense = dense / (np.sum(dense, axis=0)+.00001) # Should sum to 1.
    # Remove dustbin.
    nodust = dense[:-1, :, :]
    # Reshape to get full resolution heatmap.
    Hc = int(H / self.cell)
    Wc = int(W / self.cell)
    nodust = nodust.transpose(1, 2, 0)
    heatmap = np.reshape(nodust, [Hc, Wc, self.cell, self.cell])
    heatmap = np.transpose(heatmap, [0, 2, 1, 3])
    heatmap = np.reshape(heatmap, [Hc*self.cell, Wc*self.cell])
    ...
```



```python
class SuperPointNet(torch.nn.Module):
  ...
  def run(self, img):
    """ Process a numpy image to extract points and descriptors.
    Input
      img - HxW numpy float32 input image in range [0,1].
    Output
      corners - 3xN numpy array with corners [x_i, y_i, confidence_i]^T.
      desc - 256xN numpy array of corresponding unit normalized descriptors.
      heatmap - HxW numpy heatmap in range [0,1] of point confidences.
    """
    ...
    # --- Process descriptor.
    D = coarse_desc.shape[1]
    if pts.shape[1] == 0:
      desc = np.zeros((D, 0))
    else:
      # Interpolate into descriptor map using 2D point locations.
      samp_pts = torch.from_numpy(pts[:2, :].copy())
      samp_pts[0, :] = (samp_pts[0, :] / (float(W)/2.)) - 1.
      samp_pts[1, :] = (samp_pts[1, :] / (float(H)/2.)) - 1.
      samp_pts = samp_pts.transpose(0, 1).contiguous()
      samp_pts = samp_pts.view(1, 1, -1, 2)
      samp_pts = samp_pts.float()
      if self.cuda:
        samp_pts = samp_pts.cuda()
      desc = torch.nn.functional.grid_sample(coarse_desc, samp_pts)
      desc = desc.data.cpu().numpy().reshape(D, -1)
      desc /= np.linalg.norm(desc, axis=0)[np.newaxis, :]
    return pts, desc, heatmap
```



Summary) Feature Points and Descriptors

	Gradient-based	Intensity-based	DL-based
Feature Points	**Gradient-based** ▪ Harris ▪ GFTT (a.k.a. Shi-Tomasi) ▪ SIFT ▪ SURF	**Intensity-based** ▪ FAST	**DL-based** ▪ LIFT ▪ SuperPoint
Feature Descriptor	**Real-valued** ▪ SIFT ▪ SURF	**Binary-valued** ▪ BRIEF ▪ ORB (FAST+BRIEF)	**Real-valued (DL-based)** ▪ LIFT ▪ SuperPoint
Advantages Disadvantages	(+) Accurate (–) Slow	(+) Fast (–) Inaccurate (+) Less storage	(+) Accurate (+) Fast (–) GPU requirement

Image: pixabay

Table of Contents

- **Feature Points**
  - Gradient-based: Harris corner, GFTT corner, SIFT, SURF
  - Intensity-based: FAST
  - DL-based: LIFT, SuperPoint
- **Feature Descriptors**
  - Real-valued: SIFT, SURF (DL-based: LIFT, SuperPoint)
  - Binary-valued: BRIEF, ORB
- **Feature Matching**
  - Q) How can we associate the points across different images?
- **Feature Tracking**
  - Q) How can we associate the points across their next image?
- **Outlier Rejection**
  - Q) How can we select correctly matched points?

Feature Matching) For Real-valued Descriptors

- **Real-valued descriptors**
  - Distance measures
    - Euclidean distance: $l_2(\mathbf{d}, \mathbf{d}') = \|\mathbf{d} - \mathbf{d}'\|_2$ (↓ : similar)
    - Cosine similarity: $s_c(\mathbf{d}, \mathbf{d}') = \frac{\mathbf{d} \cdot \mathbf{d}'}{\|\mathbf{d}\| \|\mathbf{d}'\|}$ (↑ : similar)
    - Note) Matching measures can be combined or advanced.
      - e.g. The ratio of the best and second best similarity > threshold
        - It may select more distinguishable feature matching.
  - Matching algorithms
    - Brute-force search
      - Time complexity: $O(N)$ for N descriptors
    - Approximated nearest neighborhood search (ANN search)
      - Time complexity: $O(\log N)$ or less for N descriptors
      - Note) Big-ANN Competition (recent: NeurIPS 2023)

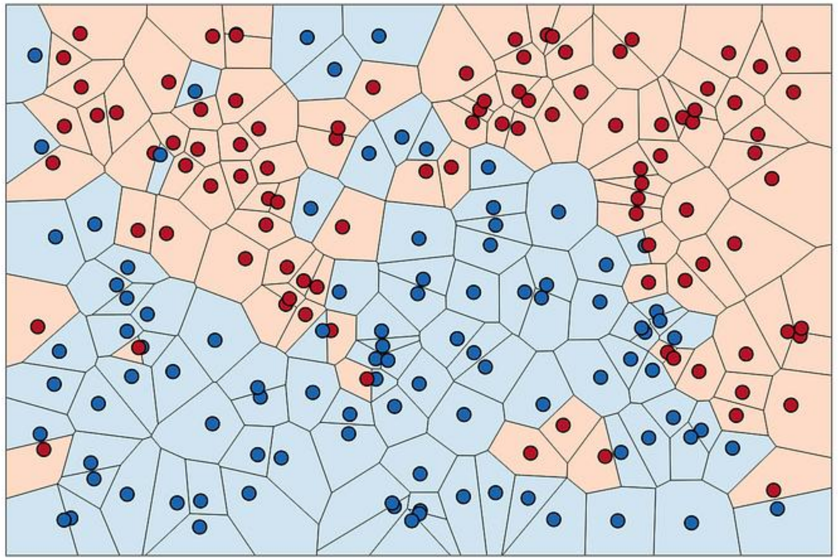


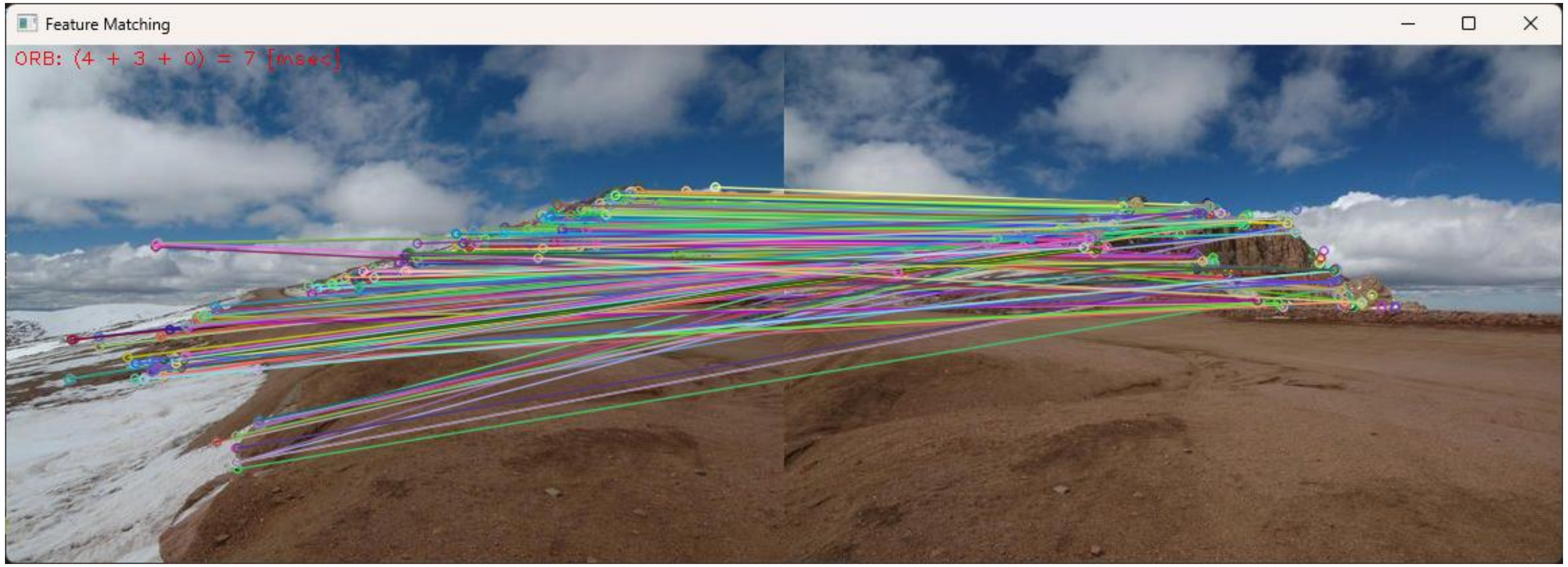
Image: George Williams, "Big-ANN-Benchmarks", Medium

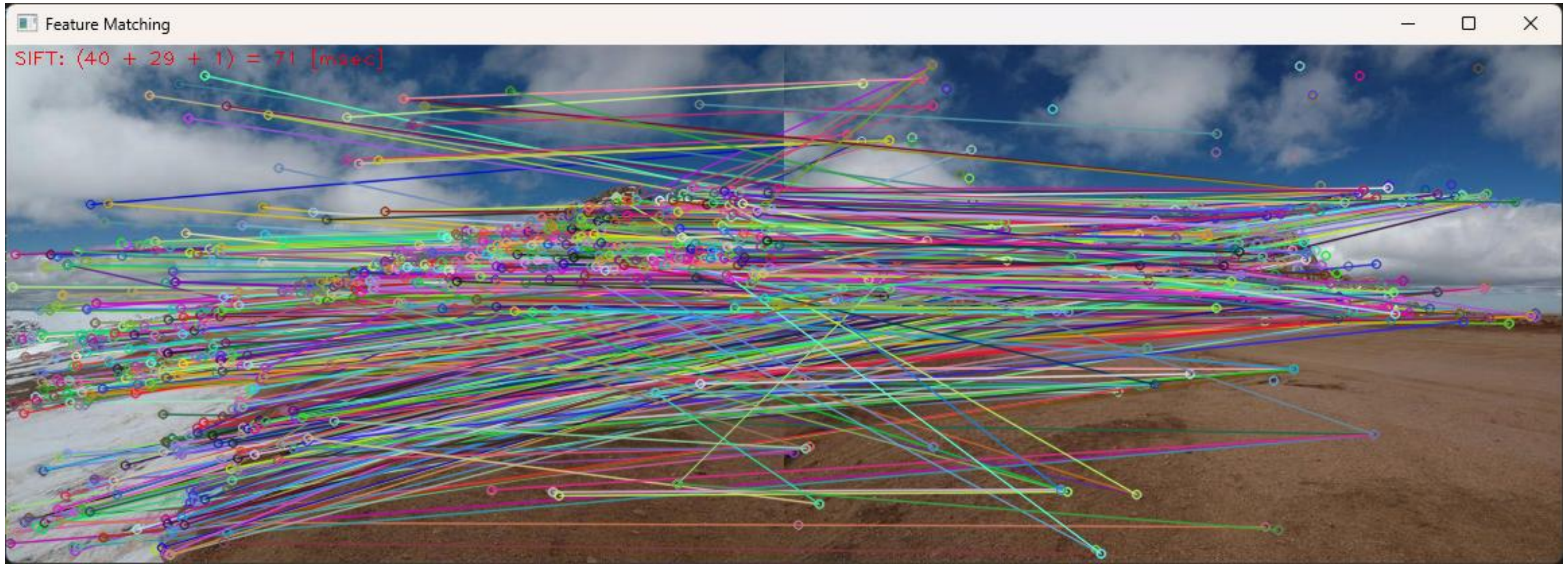
Feature Matching) For Binary-valued Descriptors

- **Binary-valued descriptors**
  - Distance measures
    - Hamming distance: $l_h(\mathbf{d}, \mathbf{d}') = \sum_i (d_i \neq d_i')$
      - e.g. 4-bit descriptors
        - $l_h(0110, 1110) = 1$ vs. (6, 14)
        - $l_h(0110, 0111) = 1$ vs. (6, 7)
        - $l_h(0110, 0101) = 2$ vs. (6, 5)
      - Note) Hamming distance is the L_1-norm with binary-valued descriptors.
  - Matching algorithms
    - Brute-force search
      - Time complexity: $O(N)$ for N descriptors

Feature Matching

- Example) **Feature matching comparison** `[feature_matching.py]`





Feature Matching

- Example) **Feature matching comparison** [feature_matching.py]

```python
# Load two images
img1 = cv.imread('../data/hill01.jpg')
img2 = cv.imread('../data/hill02.jpg')

# Instantiate feature detectors and matchers
# Note) You can specify options for each detector in its creation.
features = [
    ...
    {'name': 'FAST',    'detector': cv.FastFeatureDetector_create(),
                        'matcher' : None}, # No descriptor
    ...
    {'name': 'ORB',     'detector': cv.ORB_create(),
                        'matcher' : cv.DescriptorMatcher_create('BruteForce-Hamming')},
    {'name': 'SIFT',    'detector': cv.SIFT_create(),
                        'matcher' : cv.DescriptorMatcher_create('BruteForce')},
]
```

Feature Matching

- Example) **Feature matching comparison** [feature_matching.py]

```
# Detect feature points
keypoints1 = features[f_select]['detector'].detect(img1)
keypoints2 = features[f_select]['detector'].detect(img2)

if features[f_select]['matcher'] is not None:
    # Extract feature descriptors
    keypoints1, descriptors1 = features[f_select]['detector'].compute(img1, keypoints1)
    keypoints2, descriptors2 = features[f_select]['detector'].compute(img2, keypoints2)

    # Match the feature descriptors
    match = features[f_select]['matcher'].match(descriptors1, descriptors2)

# Show the matched image
if features[f_select]['matcher'] is not None:
    img_merged = cv.drawMatches(img1, keypoints1, img2, keypoints2, match, None)
else:
    img1_keypts = cv.drawKeypoints(img1, keypoints1, None)
    img2_keypts = cv.drawKeypoints(img2, keypoints2, None)
    img_merged = np.hstack((img1_keypts, img2_keypts))
info = features[f_select]['name’] + ...
cv.putText(img_merged, info, (5, 15), cv.FONT_HERSHEY_PLAIN, 1, (0, 0, 255))
cv.imshow('Feature Matching', img_merged)
```

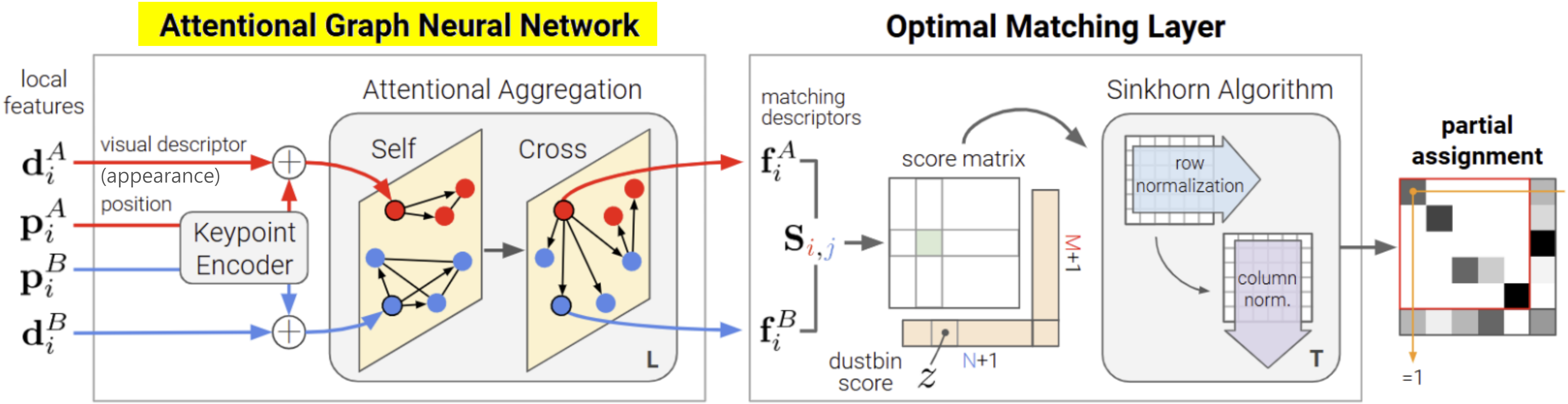
Feature Matching) SuperGlue (2020) → LightGlue (2023)

- Key idea: **Encoding feature points and descriptors using a graph neural network with attention**
  - For extreme wide-baseline image pairs in real-time on GPU
  - e.g. Relative pose estimation on the ScanNet dataset (more correct matches and less mismatches)

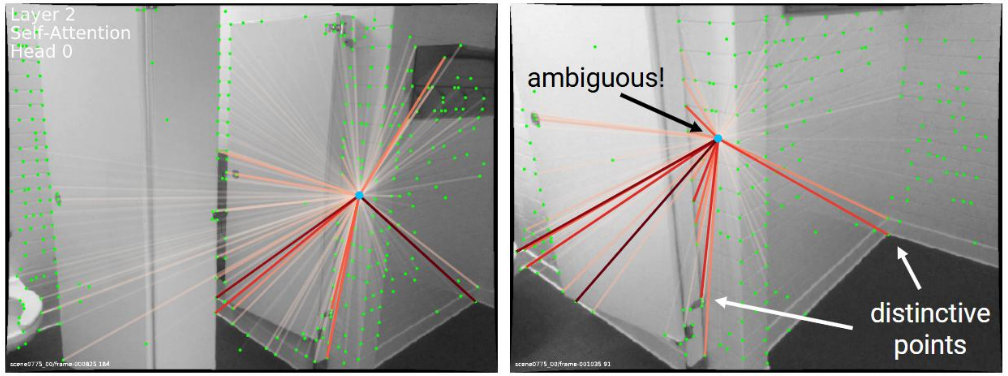
Image: Sarlin et. al., "SuperGlue: Learning Feature Matching with Graph Neural Networks", CVPR, 2020 arXiv Github (authors) Github Homepage

Feature Matching) SuperGlue (2020)

- Key idea: **Encoding feature points and descriptors using a graph neural network with attention**



Self-attention: Intra-image information flow



Cross-attention: Inter-image communication

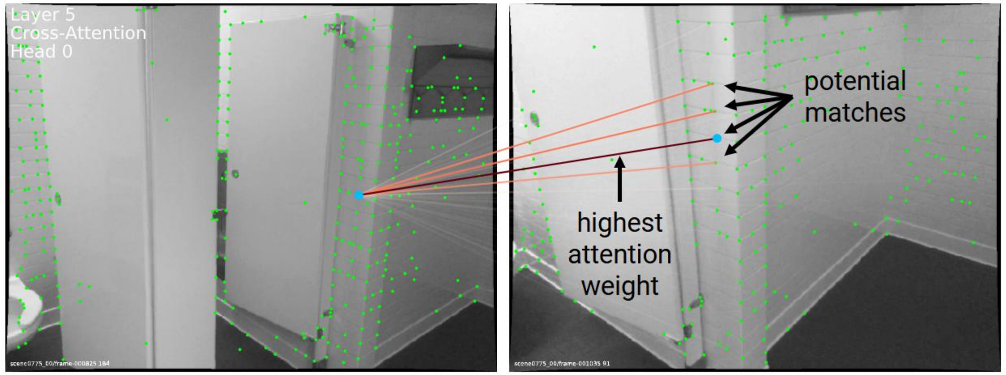
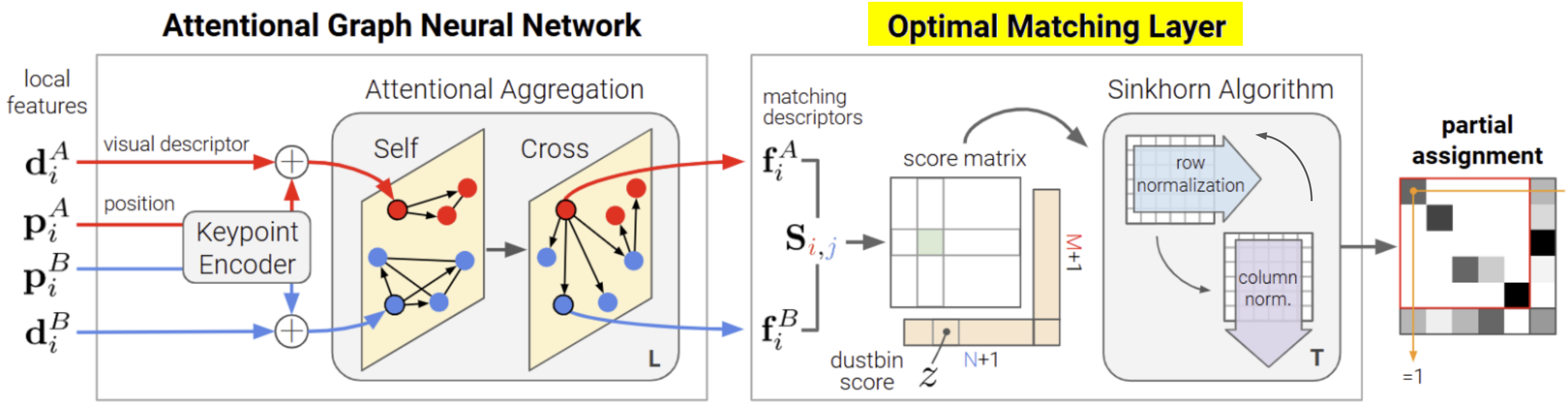


Image: Sarlin et. al., "SuperGlue: Learning Feature Matching with Graph Neural Networks", CVPR, 2020 arXiv Github (authors) Github Homepage

Feature Matching) SuperGlue (2020)

- Key idea: **Encoding feature points and descriptors using a graph neural network with attention**



– Maching score $\mathbf{S}_{i,j}$: Inner product of $\mathbf{f}_i^A$ and $\mathbf{f}_j^B$

– Sinkhorn algorithm: Differentiable & soft Hungarian algorithm

e.g. Optimal assignment problem

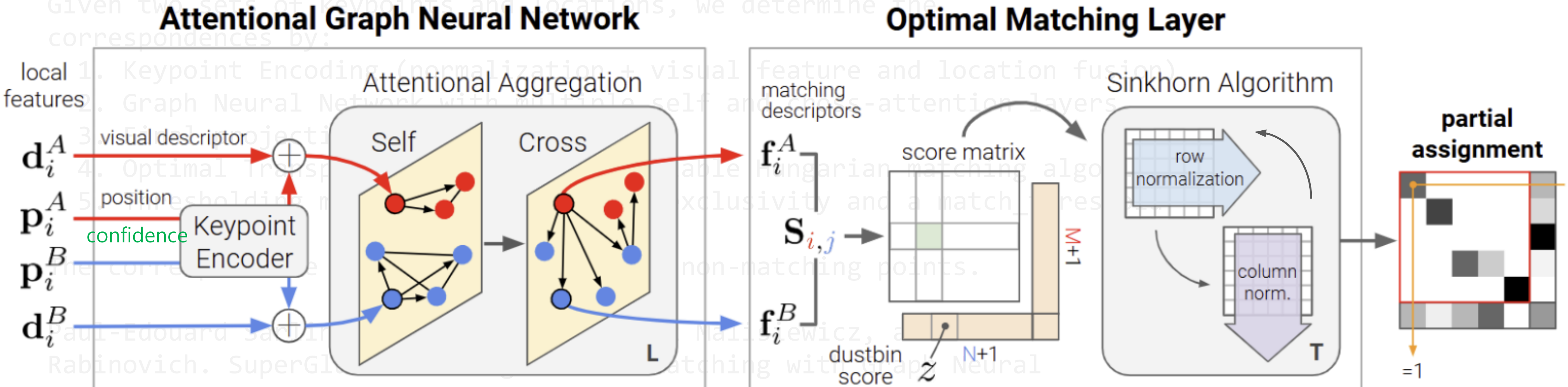
Task / Worker	Clean bathroom	Sweep floors	Wash windows
Alice	**$8**	$4	$7
Bob	$5	$2	**$3**
Carol	$9	**$4**	$8

Image: Sarlin et. al., "SuperGlue: Learning Feature Matching with Graph Neural Networks", CVPR, 2020 arXiv Github (authors) Github Homepage

Feature Matching) SuperGlue (2020)

- Example) **SuperGlue implementation** `[models/superglue.py]` @ Github (authors)

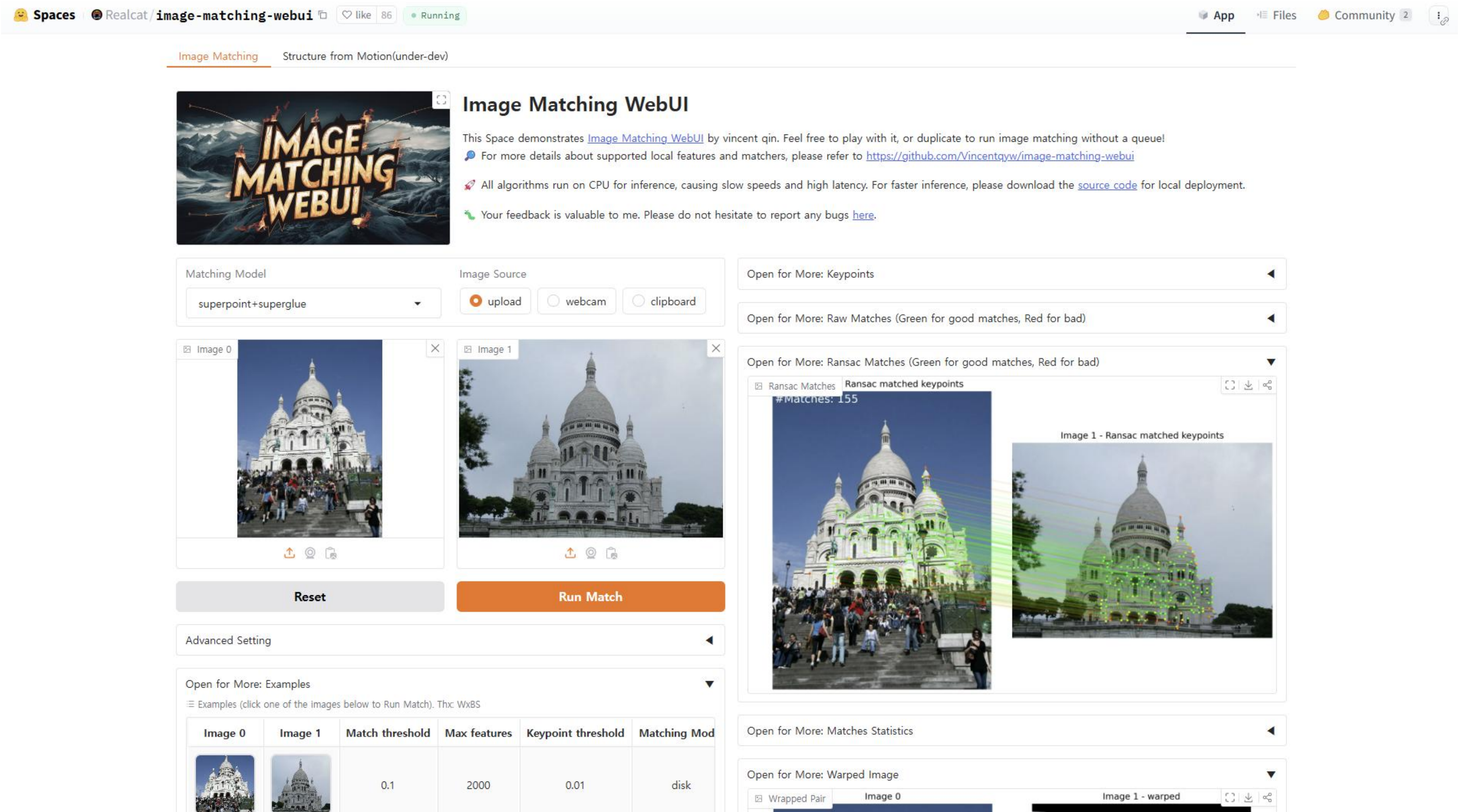
```
class SuperGlue(nn.Module):
    """SuperGlue feature matching middle-end
```



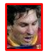

```
    """
    default_config = {
        'descriptor_dim': 256,
        'weights': 'indoor',
        'keypoint_encoder': [32, 64, 128, 256], # 4-layer CNN: [3, 32, 128, 256, 256] channels (kernel size = 1)
        'GNN_layers': ['self', 'cross'] * 9,    # 18-layer attention network + 1-layer CNN (final projection)
        'sinkhorn_iterations': 100,             # T = 100
        'match_threshold': 0.2,
    }
```

Feature Extraction and Matching

- Example) Feature matching comparison: [Image Matching WebUI]



~~Feature~~ Template Matching) For Image Patches

- **Raw image patches** (or **histogram**) can be used as descriptors.
  - Distance measures
    - SAD (sum of absolute difference): $l_1(\mathbf{d}, \mathbf{d}') = \|\mathbf{d} - \mathbf{d}'\|_1 = \sum_i |d_i - d'_i|$
    - SSD (sum of squared difference): $l_2^2(\mathbf{d}, \mathbf{d}') = \|\mathbf{d} - \mathbf{d}'\|_2^2 = \sum_i (d_i - d'_i)^2$
    - Cross-correlation: $s_{cc}(\mathbf{d}, \mathbf{d}') = \mathbf{d} \cdot \mathbf{d}' = \sum_i d_i d'_i$
    - NCC (normalized cross-correlation): $s_{NCC}(\mathbf{d}, \mathbf{d}') = \frac{\mathbf{d} \cdot \mathbf{d}'}{\|\mathbf{d}\| \|\mathbf{d}'\|}$ (~ cosine similarity)
    - ZNCC (zero-mean normalized cross-correlation): $s_{ZNCC}(\mathbf{d}, \mathbf{d}') = \frac{\bar{\mathbf{d}} \cdot \bar{\mathbf{d}}'}{\|\bar{\mathbf{d}}\| \|\bar{\mathbf{d}}'\|}$ where $\bar{\mathbf{d}} = \mathbf{d} - \mathrm{E}(\mathbf{d})$ and $\bar{\mathbf{d}}' = \mathbf{d}' - \mathrm{E}(\mathbf{d}')$
  - Matching algorithms
    - Sliding window ($\mathbf{d}$: )

SSD Cross-correlation NCC

Image: OpenCV Tutorials

Feature Tracking) Lukas-Kanade Optical Flow (1981)

- Key idea: **Finding movement of a patch whose pixel values are same**
  - Brightness constancy constraint: $I(x, y, t) = I\left(x + \Delta_x, y + \Delta_y, t + \Delta_t\right)$

$$I_x \frac{\Delta_x}{\Delta_t} + I_y \frac{\Delta_y}{\Delta_t} + I_t = 0 \quad \leftarrow \quad I\left(x + \Delta_x, y + \Delta_y, t + \Delta_t\right) \approx I(x, y, t) + I_x \Delta_x + I_y \Delta_y + I_t \Delta_t$$

$$\mathrm{A}\mathbf{x} = \mathbf{b} \quad \text{where} \quad \mathrm{A} = [I_x \quad I_y],\ \mathbf{x} = \left[\Delta_x,\ \Delta_y\right]^\top,\ \text{and}\ \mathbf{b} = [-I_t] \quad (\Delta_t = 1)$$

$$\therefore \mathbf{x} = \mathrm{A}^\dagger\, \mathbf{b}$$

- Combination: **KLT tracker**
  - GFTT (a.k.a. Shi-**T**omasi) detector + **L**ukas-**K**anade optical flow

- Advantages and disadvantages (feature tracking vs. matching)

  (+) No descriptor required (→ fast and compact)

  (–) Continuous feature tracking causes drift errors.

  (–) Not working in wide-baseline cases

  (+) Able to control matching range

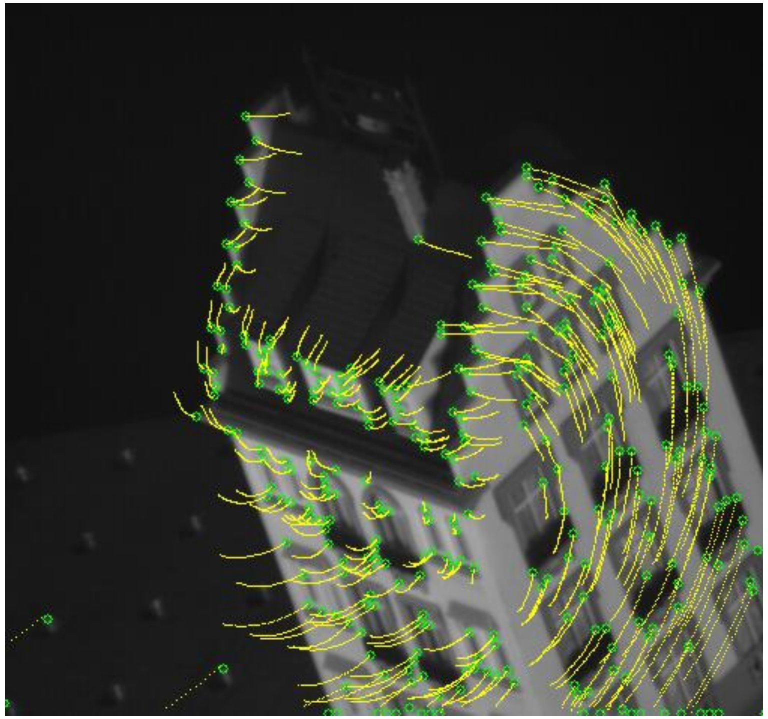


Image: Nirmal Amirthalingam, Github

Feature Tracking) Lukas-Kanade Optical Flow (1981)

- Example) **Feature matching comparison** [feature_tracking_klt.py]

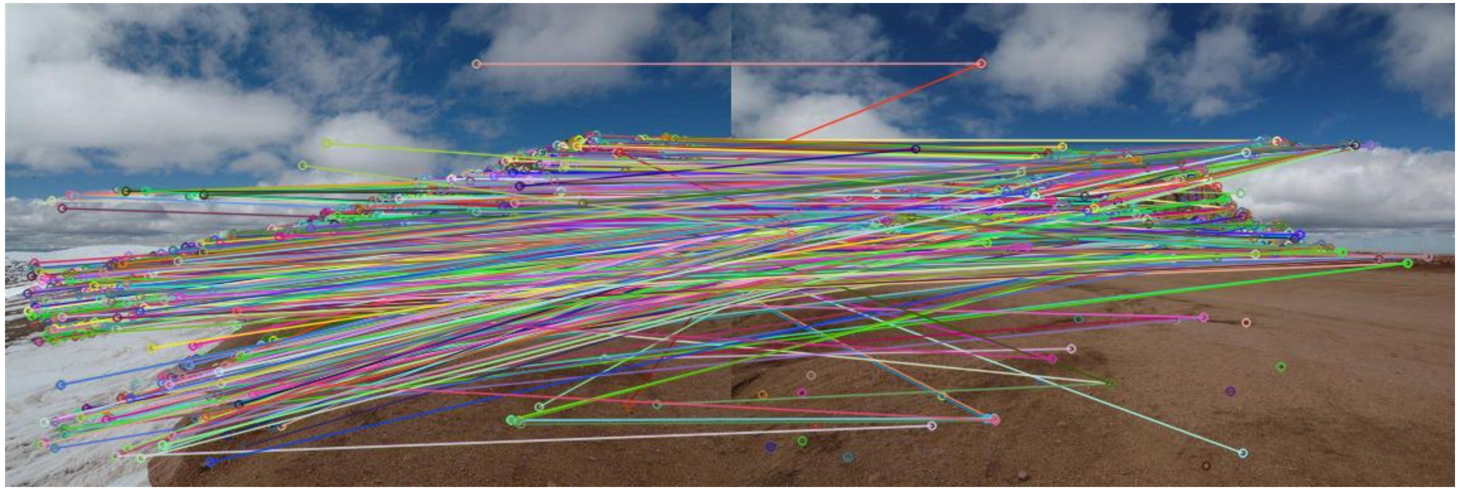
Feature Tracking) Lukas-Kanade Optical Flow (1981)

- Example) **Feature matching comparison** [feature_tracking_klt.py]

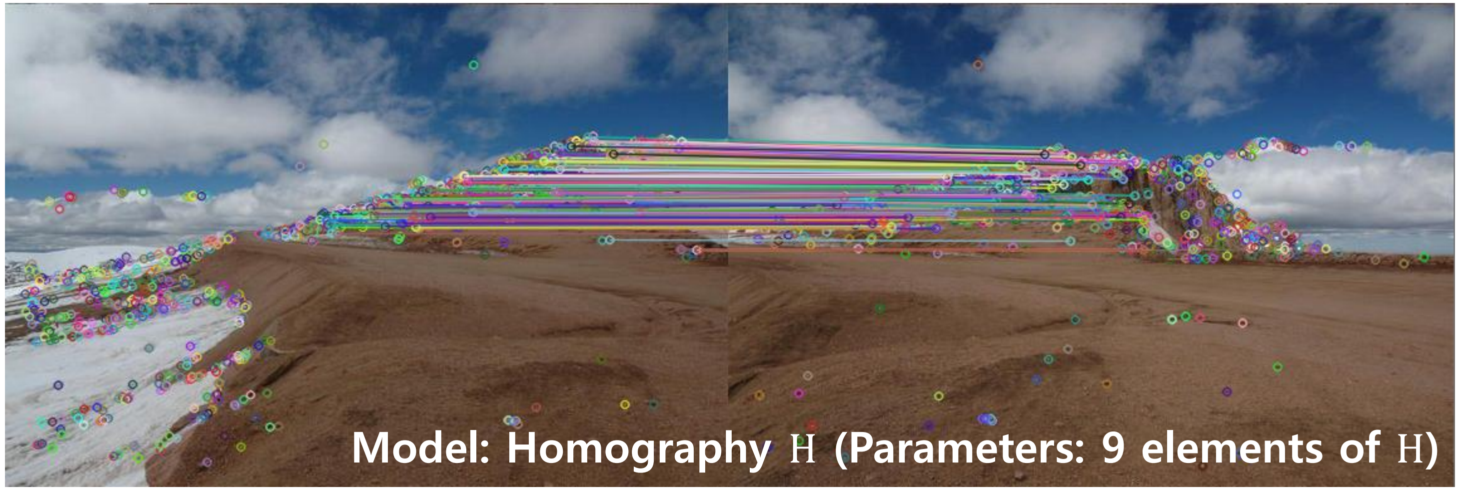
```python
# Open a video and get an initial image
video = cv.VideoCapture(video_file)
assert video.isOpened()

_, gray_prev = video.read()
if gray_prev.ndim >= 3 and gray_prev.shape[2] > 1:
    gray_prev = cv.cvtColor(gray_prev, cv.COLOR_BGR2GRAY)

# Run the KLT feature tracker
while True:
    # Grab an image from the video
    valid, img = video.read()
    if not valid:
        break
    if img.ndim >= 3 and img.shape[2] > 1:
        gray = cv.cvtColor(img, cv.COLOR_BGR2GRAY)
    else:
        gray = img.copy()

    # Extract optical flow
    pts_prev = cv.goodFeaturesToTrack(gray_prev, 2000, 0.01, 10)
    pts, status, error = cv.calcOpticalFlowPyrLK(gray_prev, gray, pts_prev, None)
    gray_prev = gray

    # Show the optical flow on the image
    if img.ndim < 3 or img.shape[2] < 3:
        img = cv.cvtColor(img, cv.COLOR_GRAY2BGR)
```

Why Outliers?

Putative feature matches (inliers + outliers)



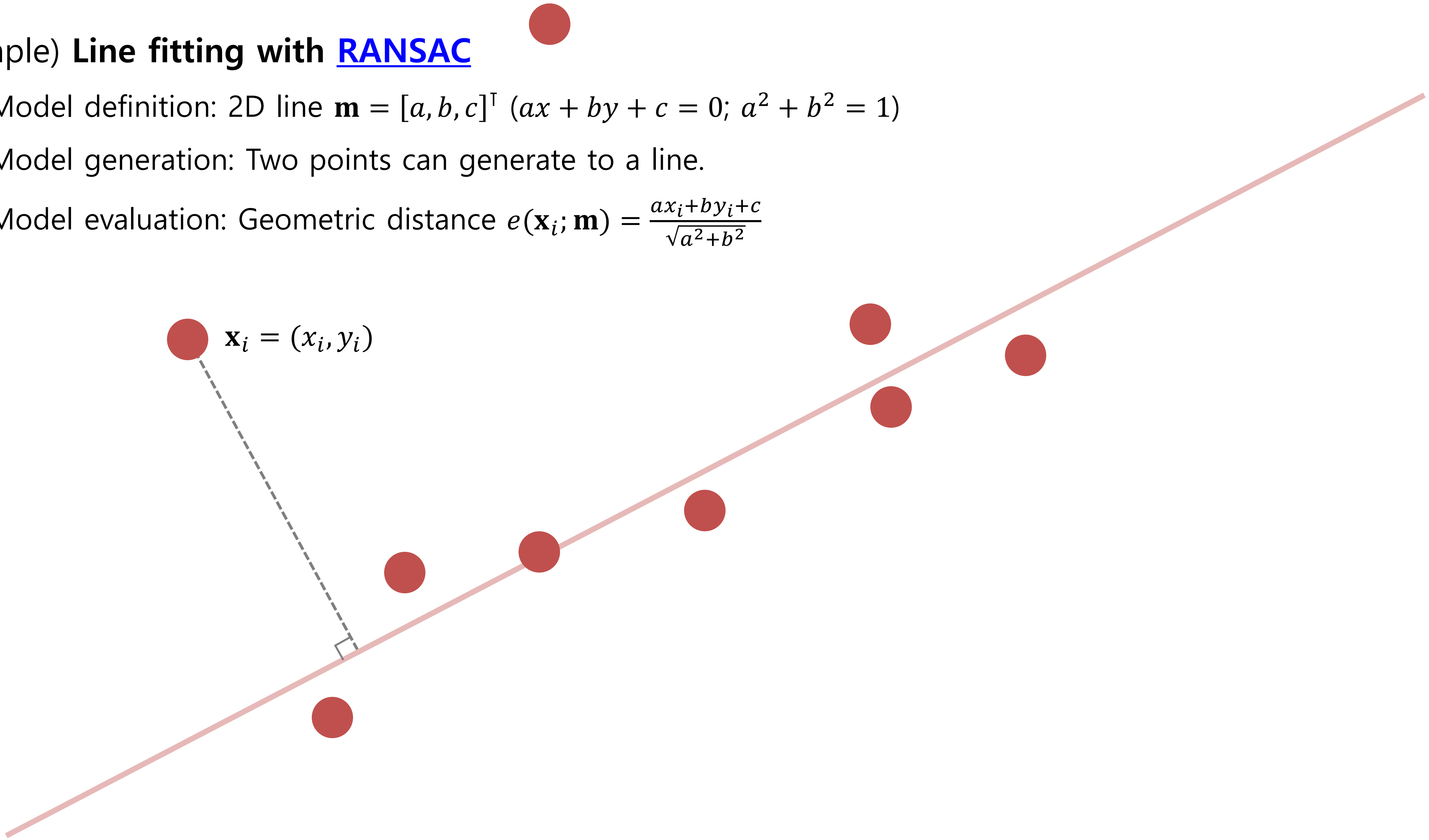
After applying RANSAC (inliers)



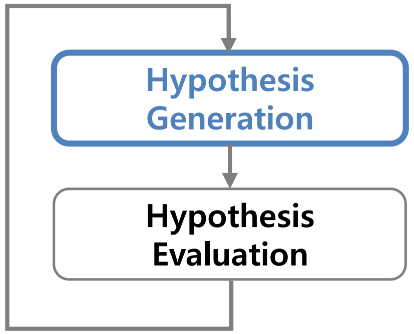
Model: Homography H (Parameters: 9 elements of H)

Outlier Rejection) RANSAC (Random Sample Consensus; 1981)

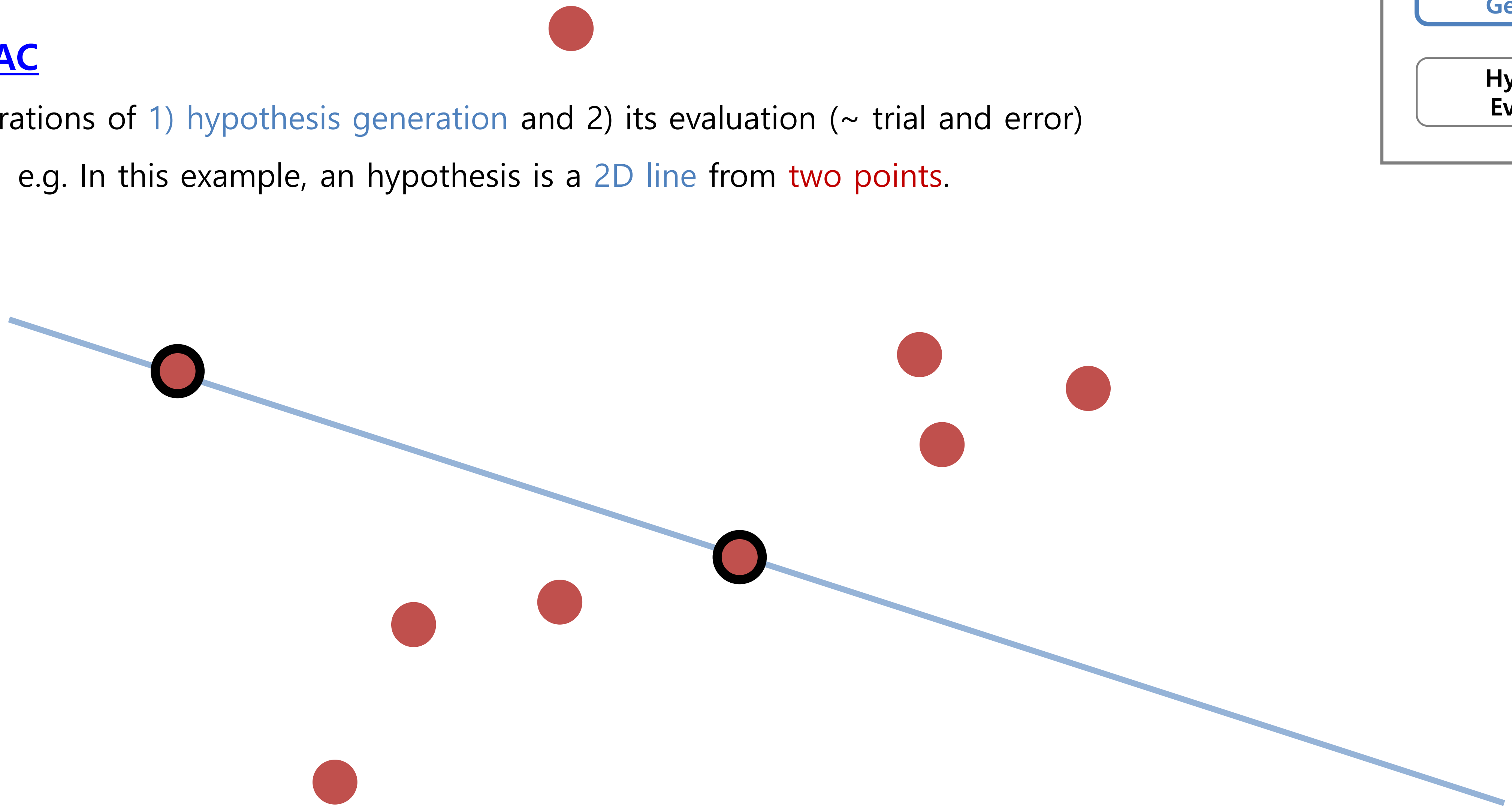
- Example) **Line fitting with RANSAC**
  - Model definition: 2D line $\mathbf{m} = [a, b, c]^\intercal$ ($ax + by + c = 0$; $a^2 + b^2 = 1$)
  - Model generation: Two points can generate to a line.
  - Model evaluation: Geometric distance $e(\mathbf{x}_i; \mathbf{m}) = \frac{ax_i + by_i + c}{\sqrt{a^2 + b^2}}$



Outlier Rejection) RANSAC (Random Sample Consensus; 1981)

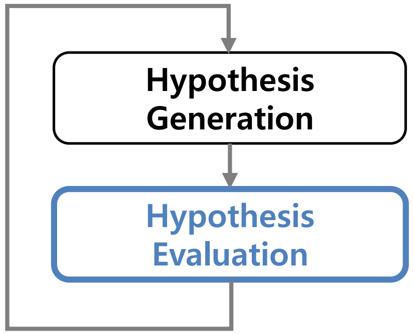


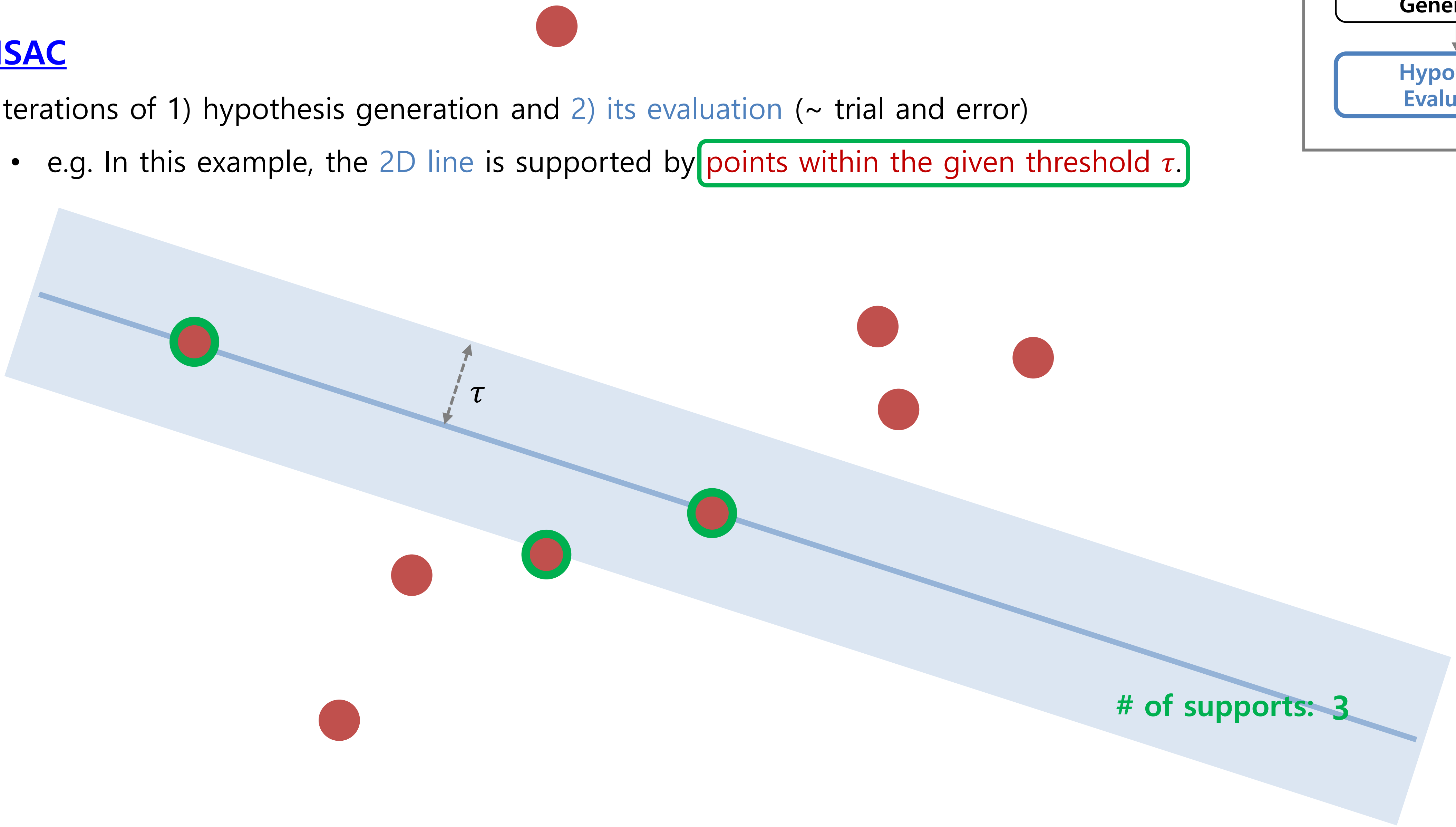
- **RANSAC**
  - Iterations of 1) hypothesis generation and 2) its evaluation (~ trial and error)
    - e.g. In this example, an hypothesis is a 2D line from two points.



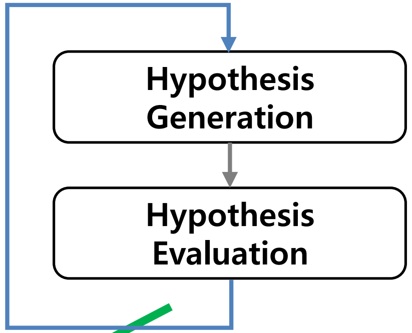
Outlier Rejection) RANSAC (Random Sample Consensus; 1981)

- **RANSAC**
  - Iterations of 1) hypothesis generation and 2) its evaluation (~ trial and error)
    - e.g. In this example, the 2D line is supported by points within the given threshold τ.

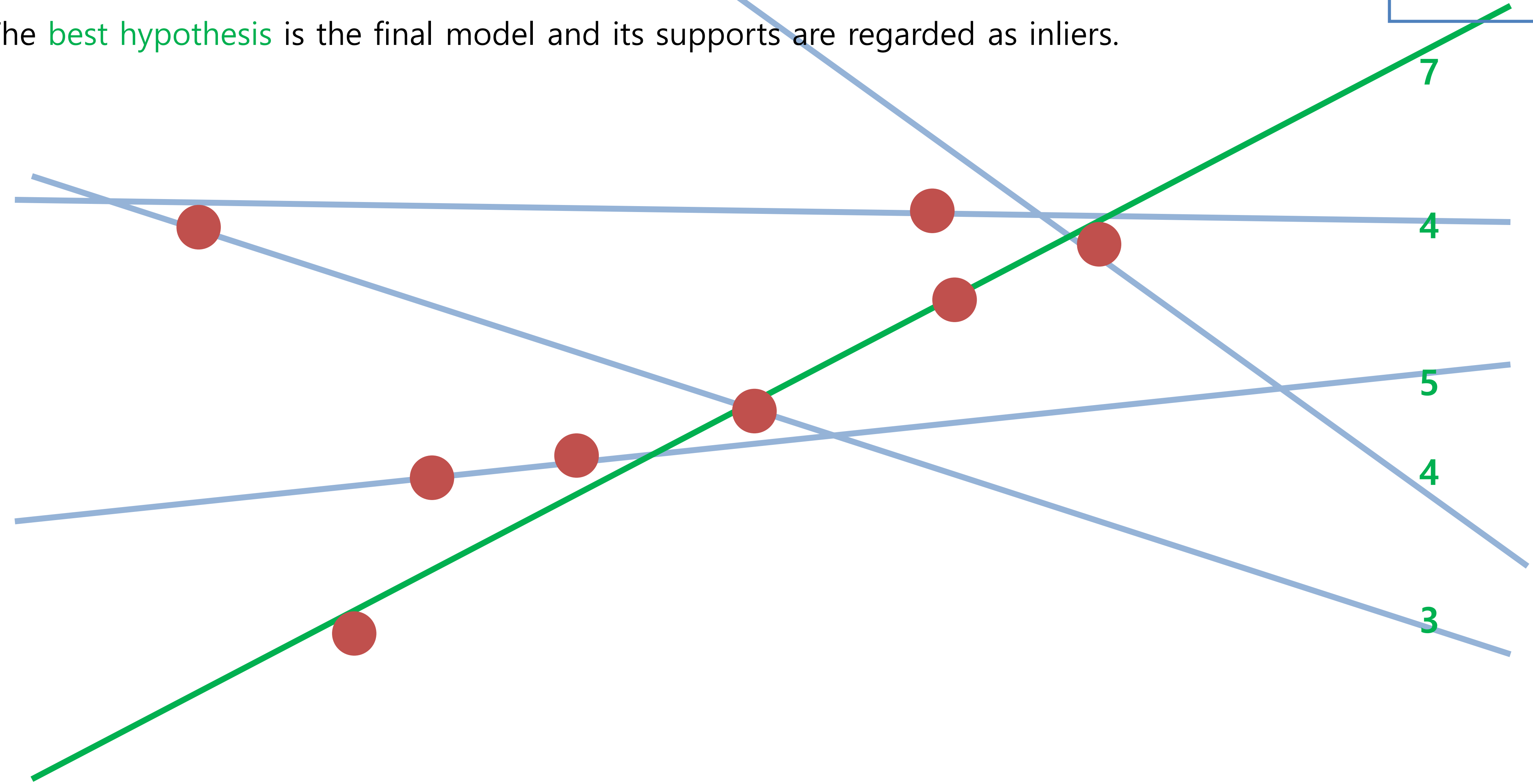




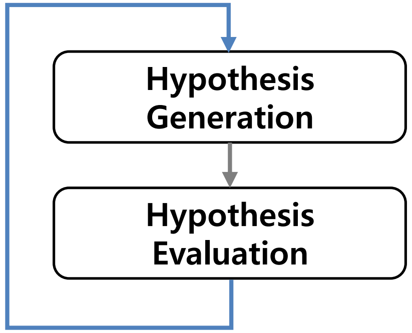
Outlier Rejection) RANSAC (Random Sample Consensus; 1981)



- **RANSAC**
  - Iterations of 1) hypothesis generation and 2) its evaluation (~ trial and error)
  - The best hypothesis is the final model and its supports are regarded as inliers.



Outlier Rejection) RANSAC (Random Sample Consensus; 1981)

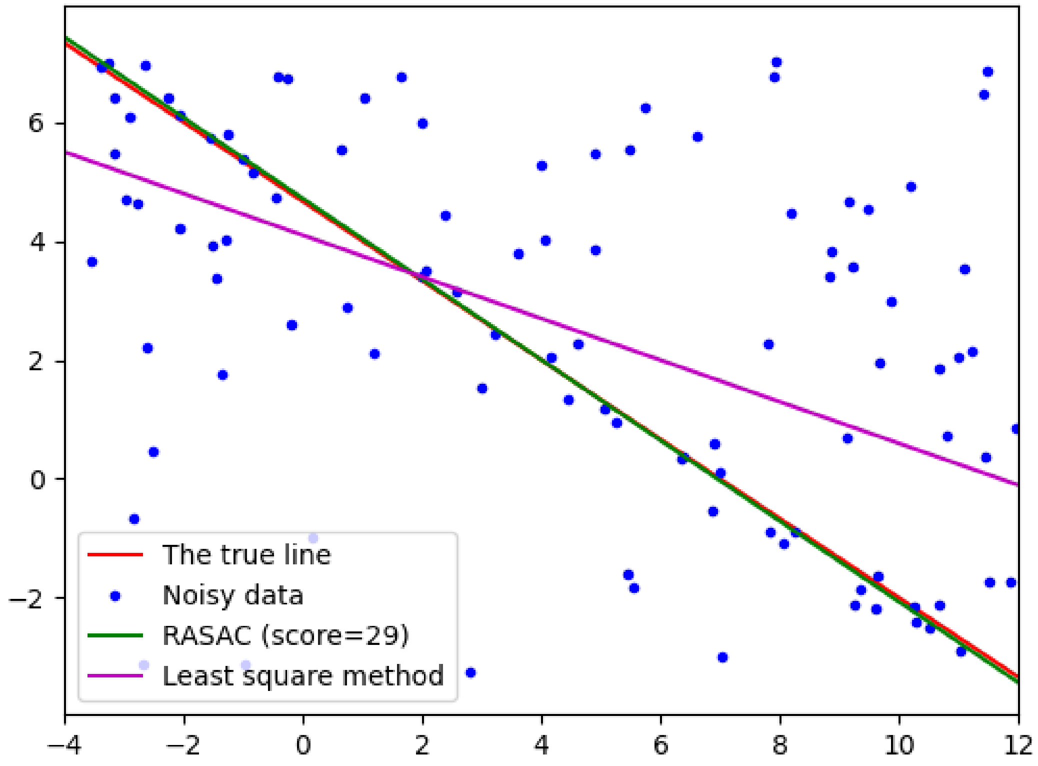


- RANSAC
  - Iterations of 1) hypothesis generation and 2) its evaluation (~ trial and error)
  - **Q) How many iterations t are required?**
    - Parameters and assumptions
      - s: Success probability (confidence level)
      - d: The number of samples for model generation
      - γ: Inlier ratio
    - Success criteria: **Selecting d samples from inliers within t iterations**
      - Failure sampling: $1 - \gamma^d$
      - Failure probability: $1 - s = \left(1 - \gamma^d\right)^t$
    - **The minimum number of iteration for success**: $t = \frac{\log(1-s)}{\log(1-\gamma^d)}$

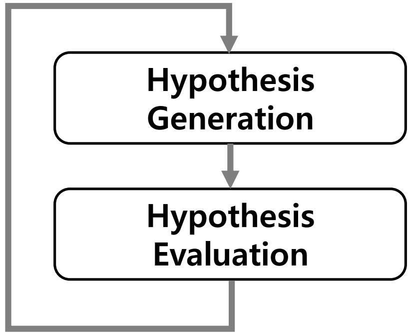
Outlier Rejection) RANSAC (Random Sample Consensus; 1981)

- Example) **Line fitting with RANSAC** `[line_fitting_ransac.py]`
  - Model definition: 2D line $\mathbf{m} = [a, b, c]^\intercal$ $(ax + by + c = 0;\ a^2 + b^2 = 1)$
  - Model generation: Two points can generate to a line.
  - Model evaluation: Geometric distance $e(\mathbf{x}_i; \mathbf{m}) = \frac{ax_i+by_i+c}{\sqrt{a^2+b^2}}$

```python
def generate_line(pts):
    # Line model: y = ax + b
    a = (pts[1][1] - pts[0][1]) / (pts[1][0] - pts[0][0])
    b = pts[0][1] - a * pts[0][0]

    # Line model: ax + by + c = 0 (a^2 + b^2 = 1)
    line = np.array([a, -1, b])
    return line / np.linalg.norm(line[:2])

def evaluate_line(line, p):
    a, b, c = line
    x, y = p
    return np.fabs(a*x + b*y + c)
```



Outlier Rejection) RANSAC (Random Sample Consensus; 1981)



- Example) **Line fitting with RANSAC** [line_fitting_ransac.py]

```python
def generate_line(pts):
    ...

def evaluate_line(line, p):
    ...

def fit_line_ransac(data, n_sample, ransac_trial, ransac_threshold):
    best_score = -1
    best_model = None
    for _ in range(ransac_trial):
        # Step 1: Hypothesis generation
        sample = random.choices(data, k=n_sample)
        model = generate_line(sample)

        # Step 2: Hypothesis evaluation
        score = 0
        for p in data:
            error = evaluate_line(model, p)
            if error < ransac_threshold:
                score += 1
        if score > best_score:
            best_score = score
            best_model = model

    return best_model, best_score
```

Outlier Rejection) RANSAC (Random Sample Consensus; 1981)

- Example) **Line fitting with RANSAC** [line_fitting_ransac.py]

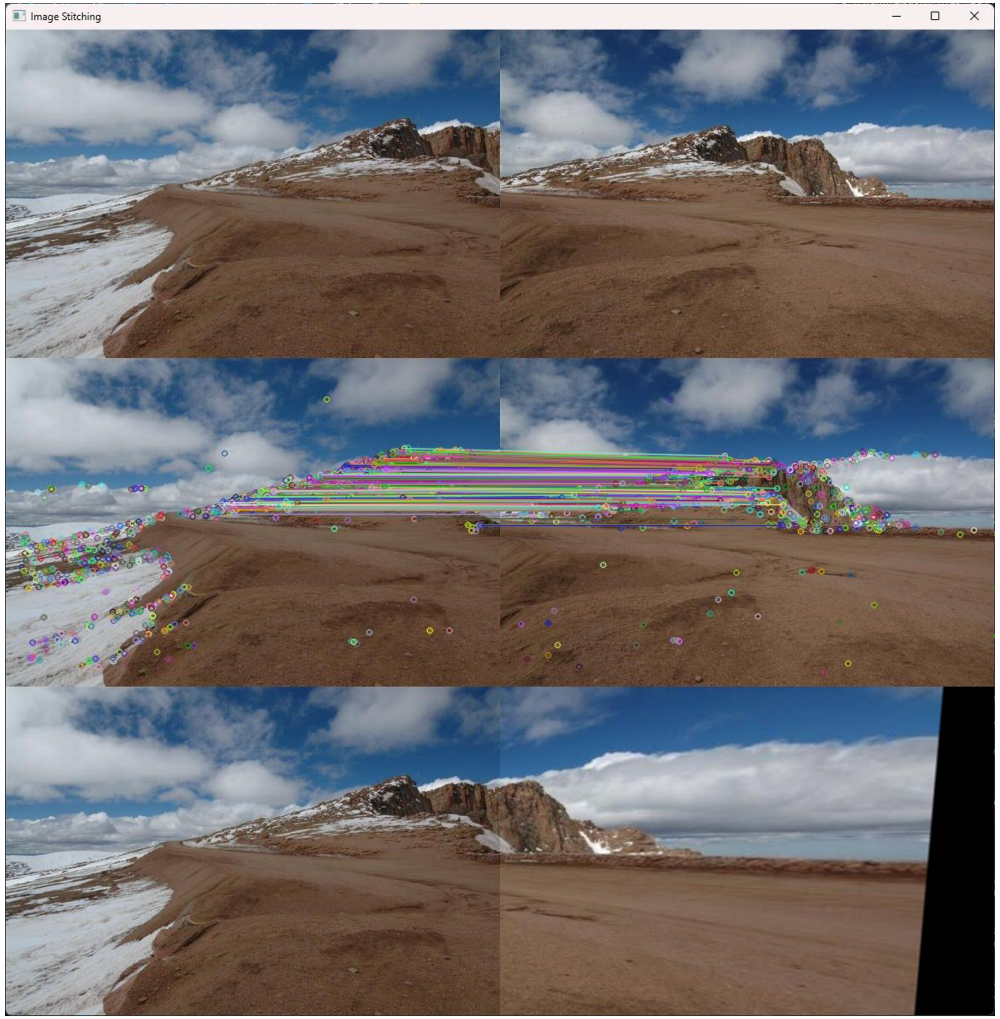
```
if __name__ == '__main__':
    true_line = np.array([2, 3, -14]) / np.sqrt(2*2 + 3*3) # The line model: a*x + b*y + c = 0 (a^2 + b^2 = 1)
    data_range = np.array([-4, 12])
    data_num = 100
    noise_std = 0.2
    outlier_ratio = 0.7

    # Generate noisy points with outliers
    line2y = lambda line, x: (line[0] * x + line[2]) / -line[1] # ax + by + c = 0 -> y = (ax + c) / -b
    y_range = sorted(line2y(true_line, data_range))
    data = []
    for _ in range(data_num):
        x = np.random.uniform(*data_range)
        if np.random.rand() < outlier_ratio:
            y = np.random.uniform(*y_range)        # Generate an outlier
        else:
            y = line2y(true_line, x)               # Generate an inlier
            x += np.random.normal(scale=noise_std) # Add Gaussian noise
            y += np.random.normal(scale=noise_std)
        data.append((x, y))
    data = np.array(data)

    # Estimate a line using RANSAC
    best_line, best_score = fit_line_ransac(data, 2, 100, 0.3) # log(1 - 0.999) / log(1 - 0.3^2) = 73

    # Estimate a line using OpenCV (for reference)
```

$$t > \frac{\log(1-s)}{\log(1-\gamma^d)} = \frac{\log(1-0.999)}{\log(1-0.3^2)} = 73$$

Review) Planar Homography

- Example) **Planar image stitching** [image_stitching.py]

```python
# Load two images
img1 = cv.imread('../data/hill01.jpg')
img2 = cv.imread('../data/hill02.jpg')

# Retrieve matching points
fdetector = cv.BRISK_create()
keypoints1, descriptors1 = fdetector.detectAndCompute(img1, None)
keypoints2, descriptors2 = fdetector.detectAndCompute(img2, None)

fmatcher = cv.DescriptorMatcher_create('BruteForce-Hamming')
match = fmatcher.match(descriptors1, descriptors2)

# Calculate planar homography and merge them
pts1, pts2 = [], []
for i in range(len(match)):
    pts1.append(keypoints1[match[i].queryIdx].pt)
    pts2.append(keypoints2[match[i].trainIdx].pt)
pts1 = np.array(pts1, dtype=np.float32)
pts2 = np.array(pts2, dtype=np.float32)

H, inlier_mask = cv.findHomography(pts2, pts1, cv.RANSAC)
img_merged = cv.warpPerspective(img2, H, (img1.shape[1]*2, img1.shape[0]))
img_merged[:,:img1.shape[1]] = img1 # Copy

# Show the merged image
img_matched = cv.drawMatches(img1, keypoints1, img2, keypoints2, match, None, None, None,
```

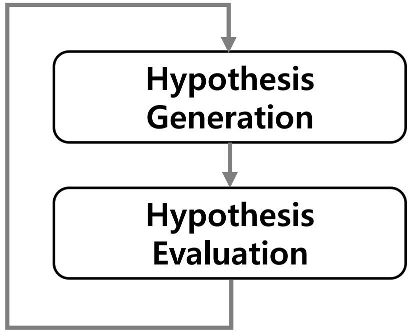


Outlier Rejection) RANSAC (Random Sample Consensus; 1981)

- Example) **Planar Homography estimation with RANSAC** [image_stitching_implement.py]

```python
from homography_estimation_implement import getPerspectiveTransform
from image_warping_implement import warpPerspective2

def findHomography(src, dst, n_sample, ransac_trial, ransac_threshold):
    ...

if __name__ == '__main__':
    # Load two images
    ...

    # Retrieve matching points
    ...

    # Calculate planar homography and merge them
    ...

    H, inlier_mask = findHomography(pts2, pts1, 4, 1000, 2) # log(1 - 0.999) / log(1 - 0.3^4) = 849
    img_merged = warpPerspective2(img2, H, (img1.shape[1]*2, img1.shape[0]))
    img_merged[:,:img1.shape[1]] = img1 # Copy

    # Show the merged image
    img_matched = cv.drawMatches(img1, keypoints1, img2, keypoints2, match, None, None, None,
                                 matchesMask=inlier_mask) # Remove `matchesMask` if you want to show all ...
    merge = np.vstack((np.hstack((img1, img2)), img_matched, img_merged))
    cv.imshow(f'Planar Image Stitching with My RANSAC (score={sum(inlier_mask)})', merge)
    cv.waitKey(0)
    cv.destroyAllWindows()
```

Outlier Rejection) RANSAC (Random Sample Consensus; 1981)



- Example) **Homography estimation with RANSAC** [image_stitching_implement.py]

```python
def evaluate_homography(H, p, q):
    p2q = H @ np.array([[p[0]], [p[1]], [1]])
    p2q /= p2q[-1]
    return np.linalg.norm(p2q[:2].flatten() - q)
```

Note) $e(\mathbf{x}, \mathbf{x}') = \|\mathrm{H}\mathbf{x} - \mathbf{x}'\|_2$

```python
def findHomography(src, dst, n_sample, ransac_trial, ransac_threshold):
    best_score = -1
    best_model = None
    for _ in range(ransac_trial):
        # Step 1: Hypothesis generation
        sample_idx = random.choices(range(len(src)), k=n_sample)
        model = getPerspectiveTransform(src[sample_idx], dst[sample_idx])

        # Step 2: Hypothesis evaluation
        score = 0
        for (p, q) in zip(src, dst):
            error = evaluate_homography(model, p, q)
            if error < ransac_threshold:
                score += 1
        if score > best_score:
            best_score = score
            best_model = model

    # Generate the best inlier mask
    best_inlier_mask = np.zeros(len(src), dtype=np.uint8)
    for idx, (p, q) in enumerate(zip(src, dst)):
```

Outlier Rejection) Least Squares Method vs. RANSAC

- **Least Squares Method**
  - Find a model while minimizing sum of squared errors, $\underset{\mathbf{m}}{\operatorname{argmin}} \sum_i e(\mathbf{x}_i; \mathbf{m})^2$
- **RANSAC**
  - Find a model while maximizing the number of supports (~ inlier candidates)
    ~ minimizing the number of outlier candidates

- **Q) Why RANSAC was robust to outliers?**
  - Problem formulation with a **loss function** ρ: $\underset{\mathbf{m}}{\operatorname{argmin}} \sum_i \rho\big(e(\mathbf{x}_i; \mathbf{m})\big)$
  - Loss function of least squares method: $\rho(x) = x^2$
  - Loss function of RANSAC: $\rho(x) = \begin{cases} 0 & \text{if } |x| < \tau \\ 1 & \text{otherwise} \end{cases}$

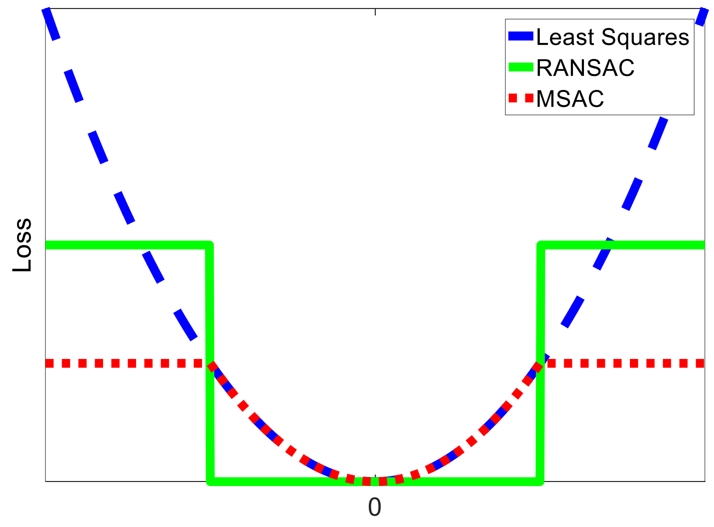
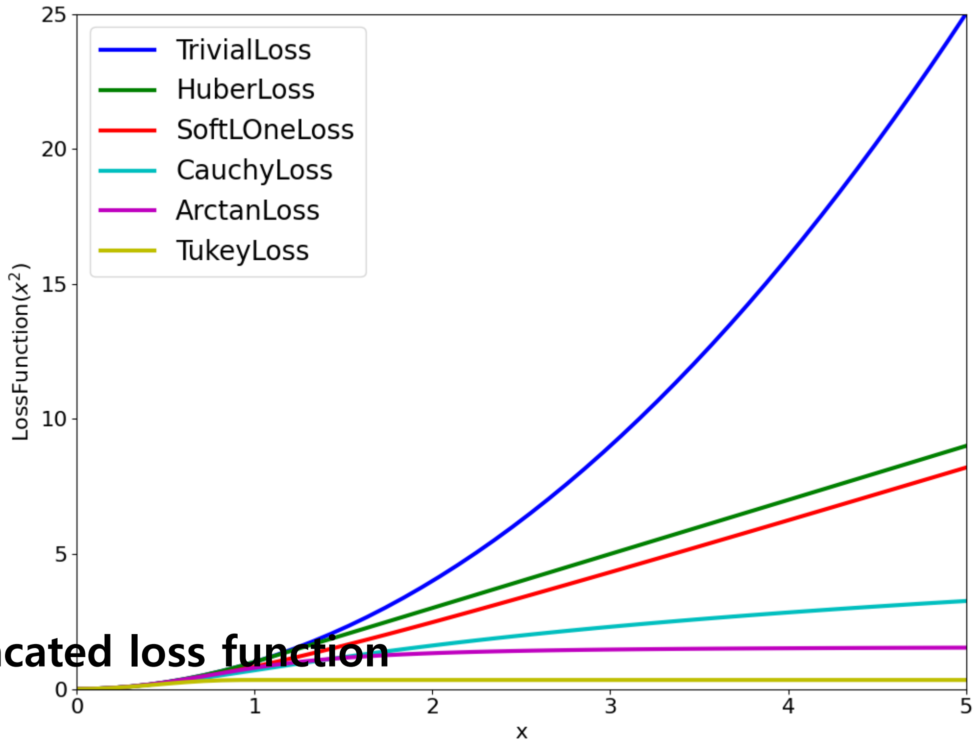


Image: Choi et al., "Performance Evaluation of RANSAC Family", BMVC, 2009 PDF

Outlier Rejection) M-estimator and MSAC



- **Robust regression**
  - Problem formulation with a **loss function** ρ: $\underset{\mathbf{m}}{\operatorname{argmin}} \sum_i \rho\big(e(\mathbf{x}_i; \mathbf{m})\big)$
  - Loss function of least squares method: $\rho(x) = x^2$
  - Loss function of RANSAC: $\rho(x) = \begin{cases} 0 & \text{if } |x| < \tau \\ 1 & \text{otherwise} \end{cases}$
- **M-estimator** (~ weighted least squares)
  - Find a model while minimizing sum of (squared) errors **with a truncated loss function**
  - Huber loss function: $\rho(x) = \begin{cases} x^2 & \text{if } |x| < \tau \\ \tau(2|x| - \tau) & \text{otherwise} \end{cases}$
  - Cauchy loss function: $\rho(x) = \ln(1 + \frac{x^2}{\tau^2})$
- **MSAC** (RANSAC implementation in OpenCV)
  - Consider <u>inliers</u> *with their quality* and regard the <u>outliers</u> *equally with limitation*
  - Loss function of MSAC: $\rho(x) = \begin{cases} x^2 & \text{if } |x| < \tau \\ \tau^2 & \text{otherwise} \end{cases}$

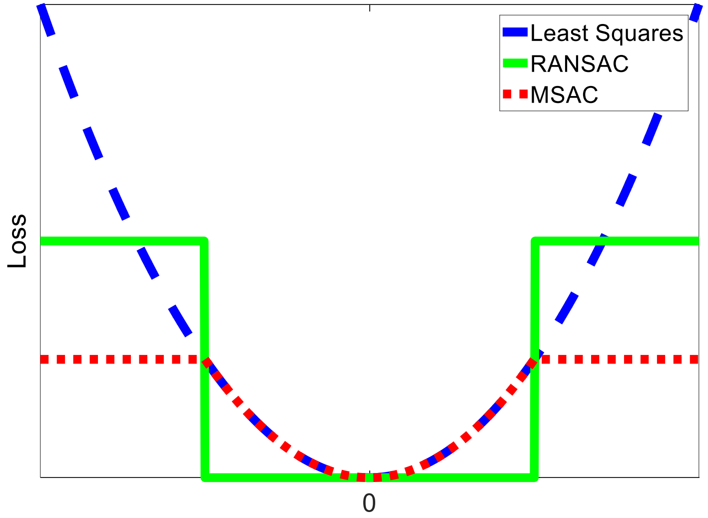


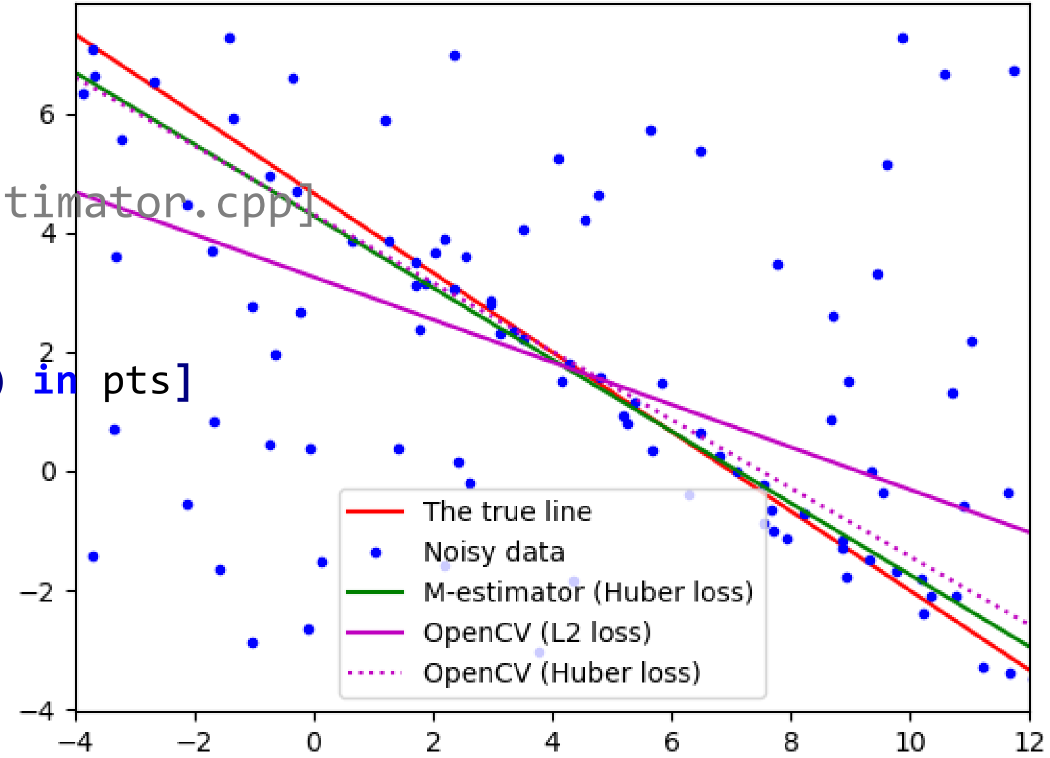
Image: Ceres Solver

Review) Solving Nonlinear Equation using Nonlinear Optimization

- **Example) Line fitting from more than two points such as $(1, 4)$, $(4, 2)$, and $(7, 1)$, using SciPy**
  - Unknown: Line parameters a, b, and c (line representation: $ax + by + c = 0$)
  - Cost function: $f(a, b, c) = \sum_i \left(\frac{ax_i + by_i + c}{\sqrt{a^2 + b^2}} \right)^2$
  - Optimizer: Gauss-Newton method (least squares)

```
import numpy as np
from scipy.optimize import least_squares

def geometric_error(line, pts):
    a, b, c = line
    err = [(a*x + b*y + c) / np.sqrt(a*a + b*b) for (x, y) in pts]
    return err

pts = [(1, 4), (4, 2), (7, 1)]
line_init = [1, 1, 0]
result = least_squares(geometric_error, line_init, args=(pts,))
line = result['x'] / -result['x'][1] # [-0.50372575, -1., 4.34823633]
```

Outlier Rejection) M-estimator



- Example) **Line fitting with M-estimator** [line_fitting_m_estimator.cpp]

```
def geometric_error(line, pts):
    a, b, c = line
    err = [(a*x + b*y + c) / np.sqrt(a*a + b*b) for (x, y) in pts]
    return err

if __name__ == '__main__':
    # Generate noisy points with outliers
    ...

    # Estimate a line using least squares with a robust kernel
    init_line = [1, 1, 0]
    result = least_squares(geometric_error, init_line, args=(data,), loss='huber', f_scale=0.3)
    mest_line = result['x'] / np.linalg.norm(result['x'][:2])

    # Estimate a line using OpenCV (for reference)
    # Note) OpenCV line model: n_y * (x - x_0) = n_x * (y - y_0)
    nnxy = cv.fitLine(data, cv.DIST_L2, 0, 0.01, 0.01).flatten()
    lsqr_line = np.array([nnxy[1], -nnxy[0], -nnxy[1]*nnxy[2] + nnxy[0]*nnxy[3]])
    nnxy = cv.fitLine(data, cv.DIST_HUBER, 0, 0.01, 0.01).flatten()
    huber_line = np.array([nnxy[1], -nnxy[0], -nnxy[1]*nnxy[2] + nnxy[0]*nnxy[3]])
```

Outlier Rejection) Applications (Model Size)

- **Bottom-up approaches (~ voting)**
  - ~~**Hough transform**~~ (not suitable for more than three model parameters)
    - **A single datum** votes multiple model candidates.
      - Note) The parameter space is maintained as a multi-dimensional histogram (discretization).
    - Score: The number of hits by data
  - **RANSAC family**
    - **A sample of data** votes a single model candidate.
    - Score: The number of supports (whose error is within threshold)
  - Note) Application examples (small-size model): Line fitting, homography estimation, relative pose estimation, PnP
- **Top-down approaches**
  - **Optimization with truncated loss functions (e.g. M-estimator)**
    - The initial model parameter moves along with the gradient of the given cost function with **whole data**.
    - Score: A cost function with a truncated loss function.
  - Note) Application examples (large-size model): Camera calibration, visual SLAM, SfM

Image: pixabay

Summary) Finding Correspondences

- **Feature Points**
  - Gradient-based: Harris corner, GFTT corner, SIFT, SURF, …
  - Intensity-based: FAST, …
  - DL-based: LIFT, SuperPoint, …
- **Feature Descriptors**
  - Real-valued: SIFT, SURF, … (DL-based: LIFT, SuperPoint, …)
  - Binary-valued: BRIEF, ORB, …
- **Feature Matching**
  - Distance measures: Euclidean distance, cosine similarity, Hamming distance, …
  - Matching methods: Brute-force search, ANN search, …
  - DL-based: SuperGlue, …
- **Feature Tracking**
  - KLT tracker = GFTT detector + LK optical flow
- **Outlier Rejection (Robust Regression)**
  - RANSAC → MSAC (inlier quality)
  - Least squares method → M-estimator (robust kernels)